Level and Inventory Control

CHAPTER 18

18.1 INTRODUCTION

Level control is extremely important for the successful operation of most chemical plants, because it is through the proper control of flows and levels that the desired production rates and inventories are achieved. Since some level processes are non-self-regulatory (i.e., unstable), automatic control is required to prevent the levels from overflowing or emptying completely when flow disturbances occur. Furthermore, the performance of some processes, such as chemical reactors, depends critically on the residence time in the vessel, which in turn depends on the level. In addition, the study of level control is helpful at this point because it emphasizes the importance of *control objectives* in controller design and tuning. Contrary to the situation with most control loops, the behavior of the manipulated variable—a flow in or out of the vessel—often is of as much importance as is the controlled variable itself! Thus, we have to modify some of the approaches developed in previous chapters to achieve the desired dynamic performance. As should be expected, these modifications are based on the principles of dynamic modelling and control system stability and performance.

In this chapter we will first review the types of inventory processes and their process dynamics. Liquid levels are used throughout this chapter, but the results are also applicable to the control of inventories of solids and gases, although the process equipment and sensors must be modified. As we will see, level is one of the few industrially important processes for which the closed-loop dynamic response can be determined analytically. Based on this analysis, the dynamic performances of standard feedback controllers are evaluated, and the tuning rules and feedback controller algorithms to meet new objectives are developed. Finally, some additional application issues, such as selecting manipulated variables for levels in series, are discussed.

18.2 ▫ REASONS FOR INVENTORIES IN PLANTS

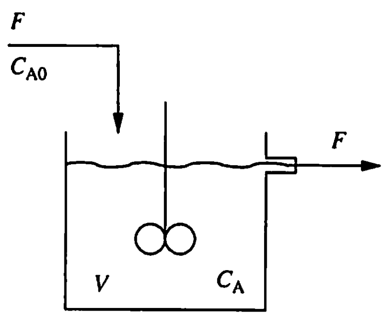
There are many good reasons to include inventories in plants. First, inventories are provided to enable plant operation to continue when some flows temporarily decrease, perhaps to zero. Some examples of periodic fluctuations in selected flows are feed material delivery, product shipping, and individual unit shutdown for maintenance. Inventories to account for these discontinuous flows can be quite large—on the order of hours or days of processing—so that plant operation can be maintained for periods when one or a few flows are zero. For example, a petroleum refinery which processes 700 m^3/h of crude oil and receives deliveries every three days requires over 50,000 m^3 of inventory and usually has much more, to store different crude oils separately and to account for delays in feed delivery.

Another important use of inventories is to ensure liquid flow to a pump. If the vessel were to empty, liquid flow would be interrupted to the pump. Many pumps cannot automatically resume flow after the flow has stopped; even worse, many pumps can be damaged if they remain in operation without flow. Therefore, a liquid inventory is required at all times. For most units, an inventory with a *holdup time* (τ_H = maximum volume divided by normal flow rate) of 5 to 10 min can attenuate normal flow variations.

Finally, inventories can be placed between a disturbance source and a sensitive unit to attenuate variation in stream properties and flow rate in input flows, so that the disturbance magnitude to the sensitive unit is significantly decreased. Vessel sizing to reduce disturbances, using frequency response principles introduced in Parts II and III, is demonstrated in the following example.

EXAMPLE 18.1.

The concentration of a feed stream to a stirred tank, C_{A0}, experiences significant variation due to upstream process operation. The liquid flow rate is 2 m^3/min, and the variation can be closely approximated as a sine wave with an amplitude of 20 g/m^3 and a period of 6 min/cycle. Analysis has determined that the disturbance cannot be reduced further in the upstream unit. The downstream chemical reactor can tolerate inlet concentration variation C_A of no more than 2.0 g/m^3. Determine the size of a well-mixed vessel to be placed before the reactor. Assume that the vessel volume is controlled at a constant value.



We begin by deriving the component material balance on the liquid in the stirred tank, as given below.

$$V\frac{dC_A}{dt} = F(C_{A0} - C_A)$$

For this example, the volume (V) and the flow rate (F) are constant; therefore, the equation is linear. We can express the balance in deviation variables from an initial steady state to give

$$\tau\frac{dC'_A}{dt} + C'_A = K_p C'_{A0} = C'_{A0} \qquad \text{where } K_p = 1 \text{ and } \tau = \frac{V}{F}$$

By taking the Laplace transform, we can determine the transfer function model.

$$\frac{C'_A(s)}{C'_{A0}(s)} = \frac{1}{\tau s + 1}$$

For this system, the time constant τ is equal to V/F. The amplitude ratio is

$$\text{AR} = \frac{|C'_A(j\omega)|}{|C'_{A0}(j\omega)|} = \frac{1}{\sqrt{\omega^2\tau^2 + 1}}$$

The value for the time constant and the volume can be calculated from these relationships:

$$\omega = 2\pi/\text{period} = (6.28 \text{ rad/cycle})/(6 \text{ min/cycle}) = 1.047 \text{ rad/min}$$

$$\text{AR} = 2/20 = 0.1$$

$$\tau = \frac{1}{\omega}\sqrt{\frac{1}{\text{AR}^2} - 1} = 9.50 \text{ min}$$

$$V = \tau F = (9.5 \text{ min})(2 \text{ m}^3/\text{min}) = 19 \text{ m}^3$$

In spite of the many helpful aspects of inventories, there are several reasons to minimize or eliminate them. First is the cost of the vessels themselves, along with the land or building space and maintenance. Second is the cost of material inventory, which is money invested in feedstock rather than distributed as profit. Third is the potential quality degradation from storing material. Finally, and often most important, is safety; the net effect of any accident can be much worse when a large inventory of flammable or hazardous material is involved.

Thus, only the minimum inventory is provided in a plant to achieve the desired dynamic operation.

As is apparent by now, control objectives play a major role in the design and tuning of feedback strategies. Levels are normally controlled by adjusting a flow in or out of the vessel. (The selection is discussed later in the chapter.) Assume that the level in Figure 18.1 is to be controlled by adjusting the flow out and that the flow in experiences flow rate disturbances. Analysis of the entire process is required to determine the control objectives, and two distinct situations commonly occur. The first, referred to as *tight* level control, is where the level is very important and variation in the manipulated flow is not of great importance; for example, this situation occurs when the vessel is a chemical reactor, with the manipulated flow going to a storage tank. The second situation, referred to as *averaging* level control, occurs when variation in the level is not important, as long as the value remains within specified limits, but the manipulated flow should not experience rapid variations with a significant magnitude. This situation occurs in controlling the level of a storage drum upstream of a critical unit. These two different control objectives are summarized in Table 18.1 with their common designations, tight and averaging level control.

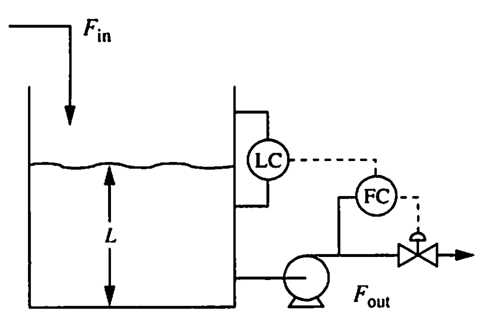


FIGURE 18.1

Typical level control system.

TABLE 18.1

Comparison of tight and averaging level control

Variable	Tight level control	Averaging level control
Controlled variable: level	Fluctuations should be reduced to a small magnitude	Fluctuations within specified limits, e.g., 20 to 80%, are allowed
Manipulated variable: flow	Fluctuations required to achieve desired level performance are accepted	Fluctuations are to be minimized, consistent with maintaining the level within limits

18.3 ▫ LEVEL PROCESSES AND CONTROLLERS

The level processes must be understood before controller algorithms can be selected. Plant vessels are built in many different shapes, such as vertical and horizontal drums and spherical and cylindrical tanks. To simplify the mathematical analysis, only cylindrical tanks with straight sides are considered in this chapter, but all results can be extended to more complex designs, although many vessels do not significantly deviate from these assumptions in their normal range of operation. Most of the level processes can be characterized by one of the four process designs shown in Figure 18.2. Each of these processes is briefly described here, and models are derived for the industrially important designs.

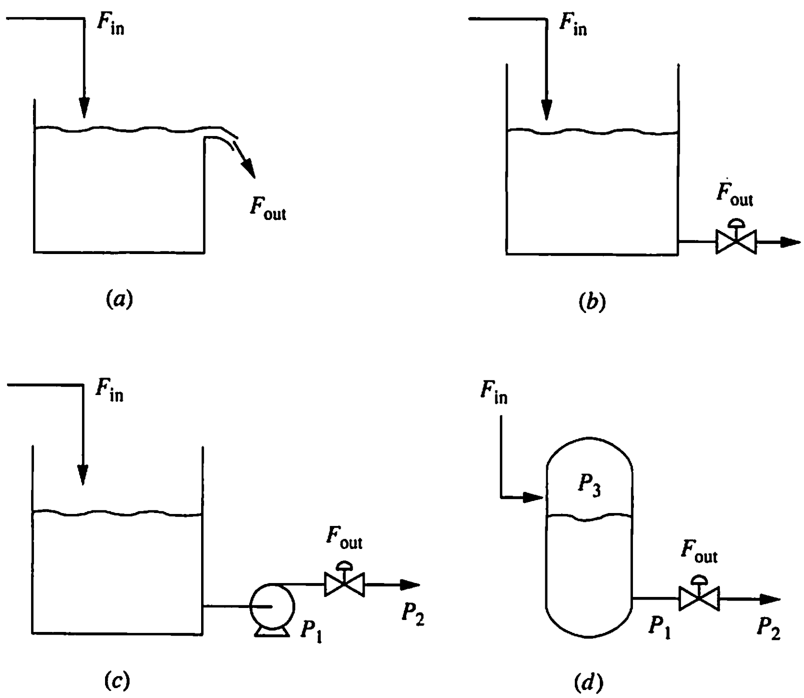


FIGURE 18.2

Various common level processes.

The overflow process in Figure 18.2*a* is seldom used in chemical plants because of its inflexibility in changing the level; however, it is used for large flows where gravity can be used as the driving force (e.g., in wastewater treatment plants). The gravity flow process in Figure 18.2*b* is not used frequently in process plants either, because it also requires a plant to flow downhill. Therefore, the process designs in Figure 18.2*a* and *b* will not be considered further in this chapter.

The level with flow out via a pump shown in Figure 18.2*c* is a very common design. The flow out depends on the valve position v and the pressure drop; here, the valve characteristic is assumed linear, so that $C_v = K$. When a pump supplies the driving force for flow, the pump outlet pressure is relatively constant; thus, the flow is independent of the level.

$$A\frac{dL}{dt} = F_{in} - F_{out} \tag{18.1}$$

$$F_{out} = K(v)\sqrt{\frac{P_1 - P_2}{\rho}} \quad \text{with } P_1 \approx \text{constant} \tag{18.2}$$

The flow from a high-pressure to a much lower-pressure system in Figure 18.2*d* also involves a nearly constant pressure drop, since the effect of the head of liquid is very small. Thus, it is independent of the liquid level.

$$A\frac{dL}{dt} = F_{in} - F_{out} \tag{18.3}$$

$$F_{out} = K(v)\sqrt{\frac{P_1 - P_2}{\rho}} \quad \text{with } P_1 = P_3 + \rho L\frac{g}{g_c} \approx P_3 \tag{18.4}$$

> The models derived in equations (18.1) to (18.4) demonstrate that the levels in Figure 18.2*c* and *d* are *non-self-regulating*, because the derivative of the level (the flows in and out) is not significantly influenced by the liquid level.

The responses of such levels without control to two common input flow disturbances are given in Figure 18.3*a* and *b*. As is apparent, the level without control can

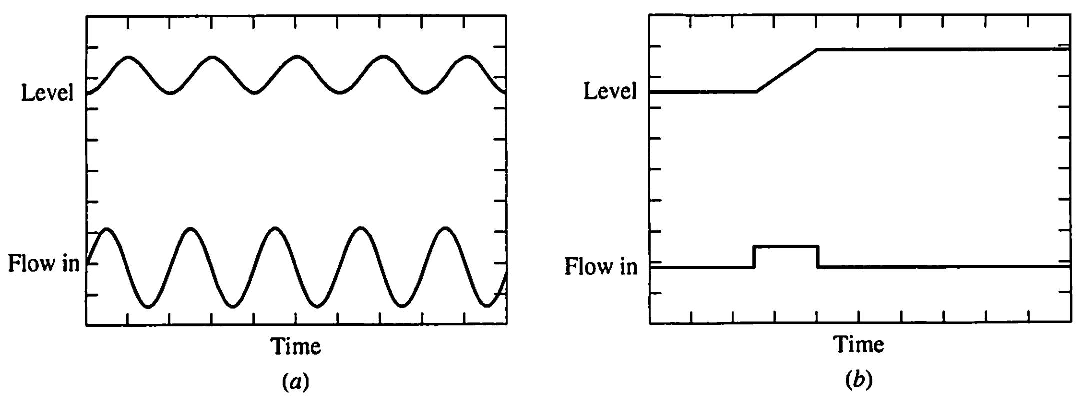


FIGURE 18.3

Response of a non-self-regulating level without control: (*a*) to sine flow variation; (*b*) to a pulse flow variation.

exceed its limits for all disturbances, depending on magnitude, and will definitely exceed limits for a step change.

Based on the open-loop responses, one would conclude that feedback control is essential. The process has no dead time and a phase lag of only 90°, indicating that feedback control would be straightforward for tight level control. This is actually the case in many systems, since the sensor and valve dynamics are usually negligible. The characteristics of several common level feedback control systems are now considered. The derivations involve the flow as the manipulated variable in a cascade structure as shown in Figure 18.1, which is essentially the same as manipulating the valve for the levels under consideration.

We begin by considering proportional-only feedback control. For the non-self-regulating process, the following derivation provides the transfer function for the closed-loop system.

$$A\frac{dL'}{dt} = F'_{\text{in}} - F'_{\text{out}} \tag{18.5}$$

with $L' = L - L_s$ and $F' = F - F_s$. Substituting the control equation ($F'_{\text{out}} = K_c(L_{\text{SP}} - L) = -K_c L'$), with $K_c < 0$ for negative feedback, A the constant cross-sectional area, and $L_s = L_{\text{SP}}$, and taking the Laplace transform yields the following transfer function:

$$\frac{L(s)}{F_{\text{in}}(s)} = \frac{1/(-K_c)}{\dfrac{A}{(-K_c)}s + 1} \tag{18.6}$$

Note that the closed-loop system is first-order, clearly self-regulating. As a result, the response of the level and the outlet flow to a step change in the inlet flow would be overdamped. As expected, the level is not necessarily controlled to its set point; the steady-state offset for a step flow disturbance (ΔF_{in}) can be determined from the final value theorem to be $\Delta F_{\text{in}}/(-K_c)$.

Next, proportional-integral control is considered. The process model in equation (18.5) is unchanged, and the controller equation becomes

$$F'_{\text{out}} = -K_c\left(L' + \frac{1}{T_I}\int_0^t L'dt'\right) \tag{18.7}$$

Substituting this expression into equation (18.5) and taking the Laplace transform yields the transfer function for the closed-loop system.

$$\frac{L(s)}{F_{\text{in}}(s)} = \frac{\left[\dfrac{T_I}{(-K_c)}\right]s}{\tau^2 s^2 + 2\tau\xi s + 1} \tag{18.8}$$

with $$\tau = \sqrt{\frac{AT_I}{(-K_c)}} \quad \text{and} \quad \xi = \frac{1}{2}\sqrt{\frac{T_I(-K_c)}{A}} \tag{18.9}$$

By applying the final value theorem, it can be shown that the system is self-regulating with zero offset for a step disturbance. The response is now second-order and can be either overdamped or underdamped, depending on the value of the damping coefficient ξ. As shown in equation (18.9), the damping coefficient depends on controller parameters K_c and T_I and the vessel area.

> **Important qualitative features of the dynamic response and the steady-state offset for the level control system depend on the process design and controller algorithm and its tuning.**

Before we determine how to match these factors to the control objectives, a modification to the linear PI controller is considered.

18.4 ☐ A NONLINEAR PROPORTIONAL-INTEGRAL CONTROLLER

Looking ahead to the application of averaging level control, we anticipate the need for an algorithm that makes small flow adjustments for small level deviations from set point and large adjusts for large deviations. Thus, a nonlinear algorithm seems appropriate. Many nonlinear modifications have been proposed; only one of the more common is discussed in this section (Shunta and Feherari, 1976). The algorithm is given as follows, and the relationship of the proportional mode between the level and manipulated flow is shown in Figure 18.4.

$$F'_{\text{out}} = -K_c\left(L' + \frac{1}{T_I}\int_0^t L'dt'\right) \tag{18.10}$$

with
$$K_c = \left\{\begin{matrix} K_{cS} & \text{when } |L'| < L'_B \\ K_{cL} & \text{when } |L'| > L'_B \end{matrix}\right\} \qquad r_K = \frac{K_{cL}}{K_{cS}}$$

Along with the integral time and gain, K_{cL}, the algorithm has two additional tuning parameters: the "break" point between the large- and small-controller-gain regions, L'_B, and the ratio of the large and small gains, r_K. Note that if the ratio is 1, the controller in equation (18.10) simplifies to a linear algorithm. If the ratio is infinity, the nonlinear controller takes no action for small deviations; that is, it has a "dead band" for an error $\pm L'_B$. The integral mode ensures that the level ultimately reaches its set point, whereas an infinite value for T_I would result in a proportional-only controller with steady-state offset.

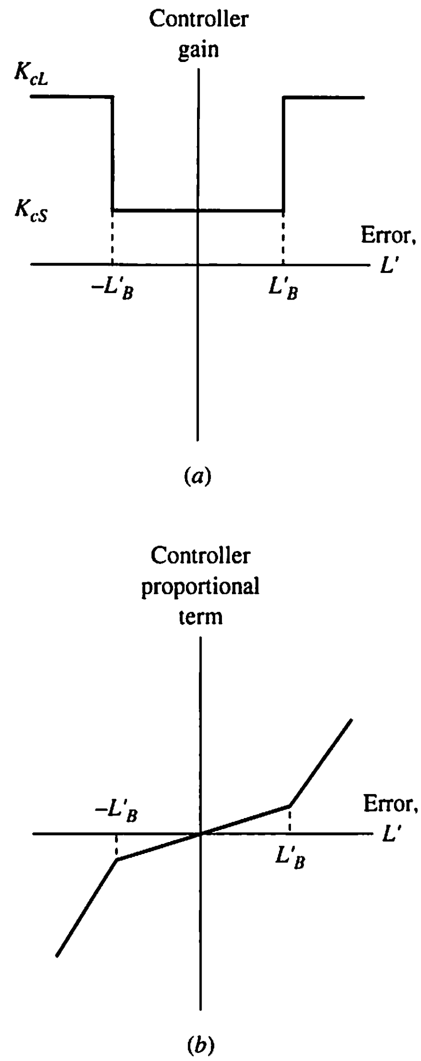


FIGURE 18.4

Graphical display of the nonlinear PI control algorithm for level control.

18.5 ☐ MATCHING CONTROLLER TUNING TO PERFORMANCE OBJECTIVES

The two sets of control objectives in Table 18.1 require different approaches, and each is presented separately in this section. The approach for determining the tuning constants for this simple process is to specify some key characteristics of the closed-loop transient response to a step flow disturbance and then to calculate tuning constants that achieve the specified characteristics. As with all tuning calculations, the resulting constants should be considered initial estimates, which can be fine-tuned based on plant performance.

Tight Level Control

We will begin by considering the case of tight level control, where the performance of the level is of greatest importance. As mentioned, the control problem is not difficult, because of the lack of dead time (or inverse response) in the process. As a result, a linear controller is adequate. The key variables used to characterize the system are the level process design and the maximum step disturbance in the uncontrolled flow. The desired transient response can be characterized by the maximum allowable level deviation in response to the disturbance and the damping coefficient ξ. A good starting value for the damping coefficient is 1.0, but the method presented here can be used for any other damping coefficient. The following expression gives the dynamic response of a level under PI control to a step flow disturbance when the damping coefficient is 1.0. With the step inlet flow, $\Delta F_{in}/s$, the expression for the level in equation (18.8) can be determined by inverting the Laplace transform using entry 6 in Table 4.1.

$$L' = \frac{\Delta F t}{A} e^{-t(-K_c)/2A} \tag{18.11}$$

The time when the maximum occurs can be determined by differentiating equation (18.11) and setting the result equal to zero, which gives a unique value of $t_{max} = 2A/(-K_c)$ because the system is not underdamped. This time can be substituted into equation (18.11) to determine the maximum level deviation for a step input.

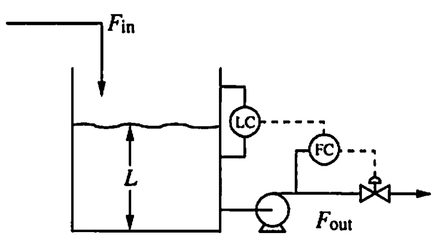
$$\Delta L_{max} = 0.736 \frac{\Delta F_{max}}{(-K_c)} \tag{18.12}$$

The tuning constants K_c and T_I can be calculated from equations (18.9) and (18.12) using specified values for the control performance: the magnitude of the disturbance, ΔF_{max}, and desired values for $\xi (= 1.0)$ and ΔL_{max}.

An alternative tuning approach, using specifications for the maximum level deviation and maximum rate of change for the manipulated flow, is given by Cheung and Luyben (1979). Their approach requires a trial-and-error solution, for which they have prepared graphical correlations.

EXAMPLE 18.2.

The level in a vessel with a volume of 20 m³, a cross-sectional area of 10 m², and a normal flow of 2 m³/min is to be controlled tightly with a PI controller. The expected maximum step change in the uncontrolled flow rate, based on plant experience, is 0.2 m³/min (i.e., 10% of normal). Tight level control requires a small level deviation, so that the maximum allowable change in the level is selected to be 0.05 m (i.e., ±2.5% of the range). Estimate the tuning constants for PI and P-only controllers.



Solution. The damping coefficient is selected to be 1.0. Using equations (18.9) and (18.12), the tuning constants for PI control are

$$K_c = \frac{-0.736 \Delta F_{max}}{\Delta L_{max}} = \frac{-0.736(0.2 \text{ m}^3/\text{min})}{0.05 \text{ m}} = -2.94 \frac{\text{m}^3/\text{min}}{\text{m}}$$

$$T_I = \frac{4\xi^2 A}{(-K_c)} = \frac{(4)(1^2)\ 10 \text{ m}^2}{2.94 \dfrac{\text{m}^3/\text{min}}{\text{m}}} = 13.6 \text{ min}$$

and, for P-only control,

$$K_c = \frac{-\Delta F_{max}}{\Delta L_{max}} = -\frac{0.20}{0.05} = -4.0 \frac{\text{m}^3/\text{min}}{\text{m}}$$

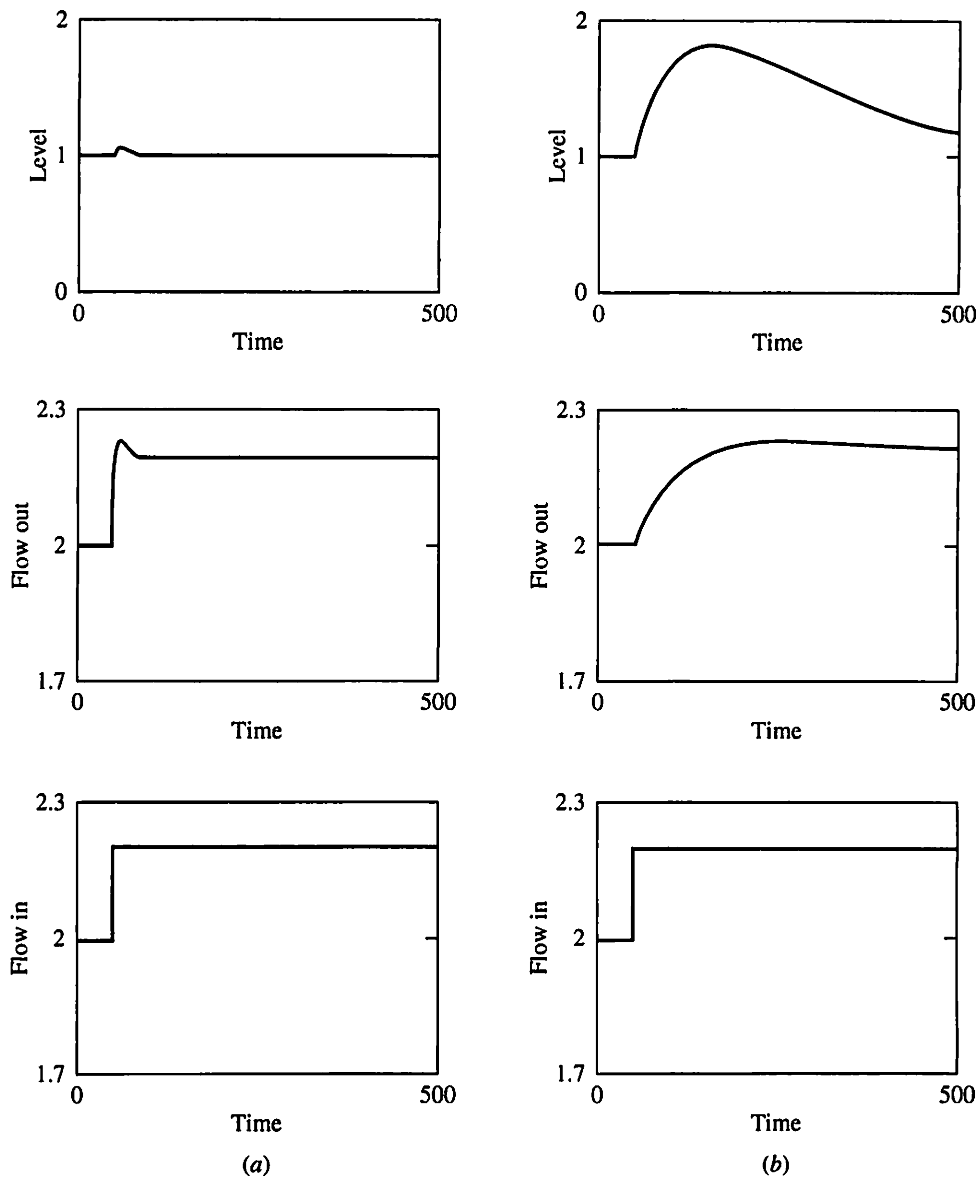


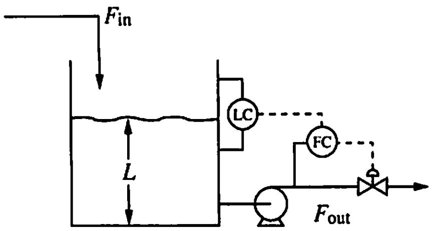
FIGURE 18.5

PI level control for Examples 18.2 and 18.3: (*a*) tight, (*b*) linear averaging.

The dynamic response for the level under tight PI control subject to the step disturbance is given in Figure 18.5*a*.

Linear Averaging Level Control

Averaging level control can be achieved with either a linear or a nonlinear controller. Both are discussed here, with the linear given first. Before presenting tuning methods, it is worth noting that averaging level control is improved by providing a large inventory (i.e., vessel volume). Thus, the performance of the averaging level system depends on the *process,* algorithm, and tuning—which is naturally true for all control systems.



The approach for the linear controller tuning is the same as for the tight control, except that the value for the allowable deviation would be much larger, to provide as much attenuation in the manipulated variable as possible.

EXAMPLE 18.3.

Calculate the tuning constants for Example 18.2 for a linear averaging level controller. All physical parameters are the same ($A = 10$ m^2, $F = 2$ m^3/min), and $\Delta F_{max} = 0.2$ m^3/min; however, the maximum level change is selected to be 0.8 m, which is ±40% of the level range, to allow inlet flow variations to be attenuated.

Solution. The same equations as in Example 18.2 are used. For PI control,

$$K_C = \frac{-0.736\Delta F_{max}}{\Delta L_{max}} = \frac{-0.736(0.2\ \text{m}^3/\text{min})}{0.8\ \text{m}} = -0.184\frac{\text{m}^3/\text{min}}{\text{m}}$$

$$T_I = \frac{4\xi^2 A}{(-K_c)} = \frac{(4)(1^2)10\ \text{m}^2}{0.184\frac{\text{m}^3/\text{min}}{\text{m}}} = 217\ \text{min}$$

and, for P-only control,

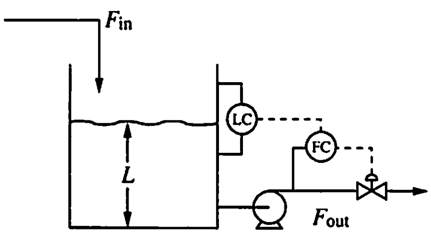
$$K_c = \frac{-\Delta F_{max}}{\Delta L_{max}} = -0.25\frac{\text{m}^3/\text{min}}{\text{m}}$$

A dynamic response for the level under averaging PI control subject to the step disturbance is given in Figure 18.5*b*. The slower response of the flow out is obvious, and the maximum rate of change of the manipulated flow is about 1/15 the value for the tight level control response, which was achieved with the same vessel and control algorithm through modified tuning.

Nonlinear Averaging Level Control

The nonlinear controller has two additional parameters to specify. With proper values for these parameters, the nonlinear controller can provide better performance (i.e., make smaller manipulations) when the system experiences frequent, small flow disturbances. The value of L'_B is selected to be smaller than the maximum level deviation but to be larger than most level variations experienced in normal operation. The value for the gain ratio is selected to provide small corrections for the small deviations; a value of 20 is usually a good starting point. To simplify the calculations for the initial estimates, the proportional gain is calculated so that the proportional term *alone* can correct for the largest expected flow disturbance. The proportional term can be calculated as follows by conforming to Figure 18.4*b*:

$$\Delta F_{max} = -K_{cS}L'_B - K_{cL}(\Delta L_{max} - L'_B) = \left(\frac{L'_B}{r_K} + \Delta L_{max} - L'_B\right)(-K_{cL}) \tag{18.13}$$



Then the integral time is calculated so that the damping coefficient is 1.0 for the small-gain region, which ensures that the damping coefficient is greater than one in the large-gain region.

EXAMPLE 18.4.

Calculate the tuning constants for the averaging level control objective and process in Example 18.3 with a nonlinear averaging controller.

Solution. The nonlinear controller requires two additional parameters. The guidelines suggest that $r_K = 20$, and we select L'_B to be relatively large, to provide small outlet flow variations for most inlet flow oscillations. Thus, $L'_B = 0.7$ m, which is ±35% of the level range. For the PI controller,

$$A = 10 \text{ m}^2 \qquad F = 2 \text{ m}^3/\text{min} \qquad \Delta F_{max} = 0.2 \text{ m}^3/\text{min}$$

$$K_{cL} = \frac{-\Delta F_{max}}{\frac{L'_B}{r_K} + \Delta L_{max} - L'_B} = \frac{-0.2 \text{ m}^3/\text{min}}{\frac{0.7\text{m}}{20} + 0.1 \text{ m}} = -1.48 \frac{\text{m}^3/\text{min}}{\text{m}}$$

$$K_{cS} = \frac{K_{cL}}{20} = -0.074 \frac{\text{m}^3/\text{min}}{\text{m}}$$

$$T_I = \frac{4\xi^2 A}{(-K_{cL}/r_K)} = \frac{(4)(1^2)10 \text{ m}^2}{0.074 \text{ m}^3/\text{min}} = 540 \text{ min}$$

Now that we have tuned the linear and nonlinear controllers, it is worthwhile comparing their performance for a periodic input disturbance, because plants often experience such variation. The responses to sine disturbances are given in Figure 18.6*a* through *c* for the tunings determined in Examples 18.2 through 18.4, with the input flow disturbance a sine with magnitude 0.2 m^3/min and period of 80 min. The results in Figure 18.6*a* demonstrate the performance of the tight level controller, which maintains the level close to its set point but has a large maximum rate of change in the output flow, 1.8×10^{-2} (m^3/min)/min. Recall that it is not possible to achieve tight level control with small flow manipulations simultaneously.

> **A linear PI controller provides excellent performance when tight level control is required. The alternative design, using a proportional-only controller with a high controller gain, is also acceptable.**

The performance for averaging level control demonstrates that both linear and nonlinear approaches provide flow attenuation; in other words, the manipulated flow varies substantially less than the inlet flow. The response for the linear averaging PI controller is given in Figure 18.6*b*, which demonstrates the smaller variability in the manipulated flow [the maximum rate of change is 0.40×10^{-2} (m^3/min)/min], and a larger variability in level. The response for the nonlinear averaging PI controller is given in Figure 18.6*c*, which demonstrates the even smaller variability in the manipulated flow (the maximum rate of change is 0.16×10^{-2} (m^3/min)/min) and a yet larger variability in level. Note that the nonlinear averaging level controller reduced the maximum rate of change of the manipulated flow by an order of magnitude when compared with the tight controller for the *same inventory volume.*

> **The nonlinear level controller is preferred for averaging control when the flow variations and vessel volume are such that the level remains within $\pm L'_B$ for most of the time.**

The level algorithms and tuning in this section have provided the flexibility to use the existing inventory to the greatest advantage. However, acceptable performance for averaging level control requires sufficient inventory; therefore, determining the proper inventory is addressed in the next section.

18.6 ☐ DETERMINING INVENTORY SIZE

Naturally, the control performance is influenced by the vessel holdup time, so that an important task of the engineer is to determine inventory sizes when designing or modifying the plant. Given the flow rate disturbance, the performance specification, and the controller tuning method, the holdup time can be determined

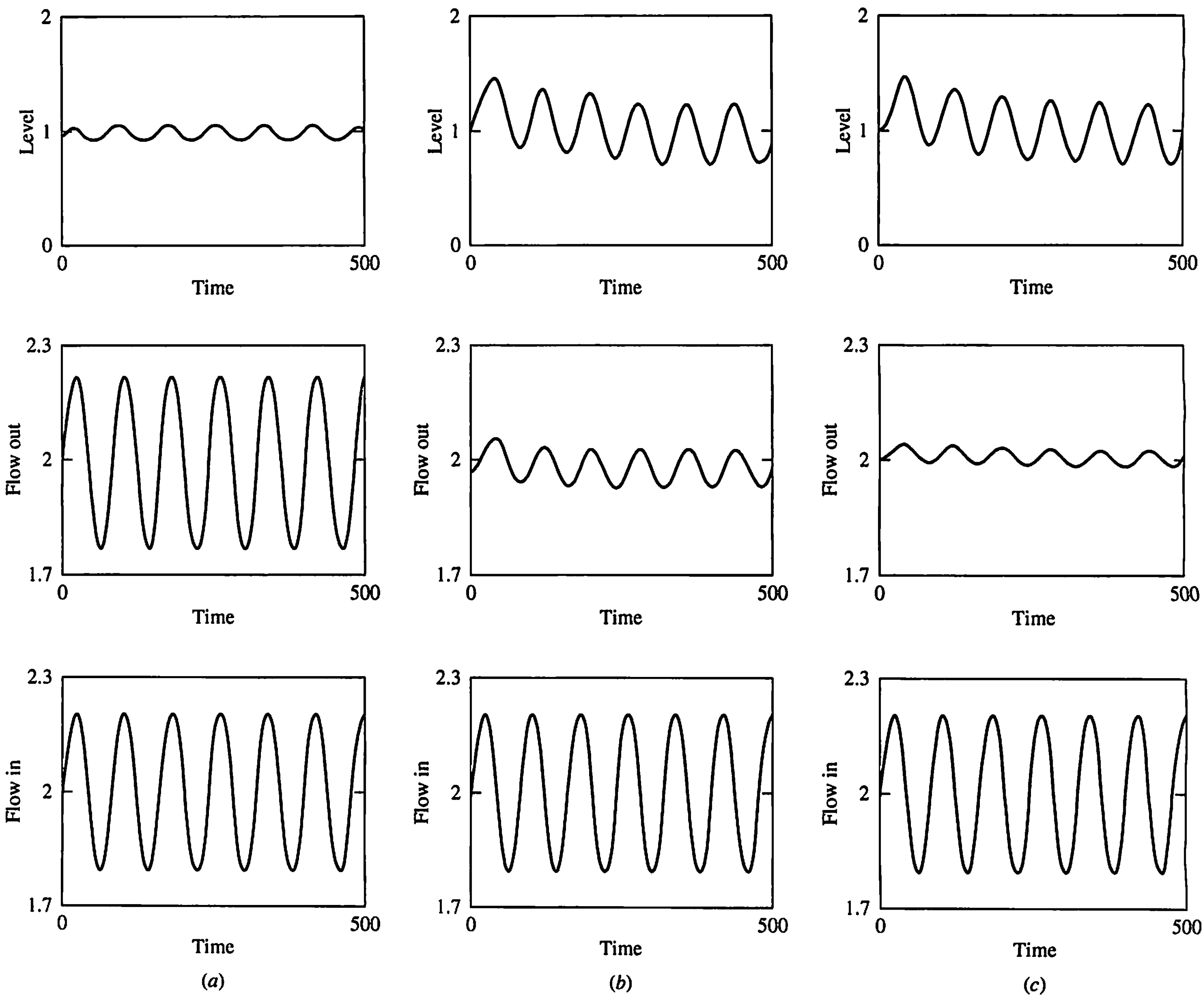


FIGURE 18.6

Level control for an input sine flow disturbance: (*a*) tight PI control with tuning from Example 18.2; (*b*) linear averaging control with tuning from Example 18.3; (*c*) nonlinear averaging control with tuning from Example 18.4.

using the results from previous sections. For a step disturbance, the calculations would involve the relationships already derived and used in tuning calculations to determine the volume required to maintain the level within $\pm\Delta L_{max}$ and the maximum rate of change of the manipulated variable at or below a specified value. It is assumed that the damping coefficient should be 1.0, although the approach can be adapted for other values.

The calculation of the inventory size can be performed in a noniterative manner by using the analytical expression of the manipulated flow to a step change in the in flow. First, the transfer function relating the flows in and out is derived using equation (18.8) and the PI controller transfer function:

$$\frac{F_{out}(s)}{F_{in}(s)} = \frac{L(s)}{F_{in}(s)}\frac{F_{out}(s)}{L(s)} = \frac{\frac{T_I}{-K_c}s}{\tau^2 s^2 + 2\xi\tau s + 1}\left[-K_c\left(1+\frac{1}{T_I s}\right)\right] = \frac{T_I s + 1}{\tau^2 s^2 + 2\xi\tau s + 1} \tag{18.14}$$

Then the step input is substituted ($F_{in}(s) = \Delta F_{in}/s$) and the inverse Laplace transform is determined from entry 8 in Table 4.1 to give

$$F'_{out}(t) = \Delta F_{in}\left[1 + \left(\frac{T_I - \tau}{\tau^2}t - 1\right)e^{-t/\tau}\right] \tag{18.15}$$

The derivative of the flow rate can then be taken to give

$$\frac{dF_{out}}{dt} = \Delta F_{in}\left[\left(\frac{T_I - \tau}{\tau^2} - \frac{T_I - \tau}{\tau^3}t + \frac{1}{\tau}\right)e^{-t/\tau}\right] \tag{18.16}$$

It is clear from this result (noting that $T_I > \tau$ for the tuning selected) that the maximum rate of change occurs at $t = 0$. Setting $t = 0$ and substituting the value of τ from equation (18.9) gives

$$\left.\frac{dF_{out}}{dt}\right|_{max} = \left(\frac{\Delta F_{in}}{A}\right)(-K_c) \tag{18.17}$$

The value of the controller gain from equation (18.12) can be substituted to give

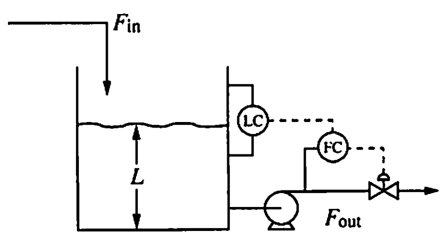
$$\left.\frac{dF_{out}}{dt}\right|_{max} = \frac{0.736(\Delta F_{in})^2}{A(\Delta L_{max})} \tag{18.18}$$

The product $A(\Delta L_{max})$ represents the allowable variability in the inventory above (or below) the set point. If the level is allowed to vary $\pm 40\%$, $A(\Delta L_{max}) = 0.40\ V$. Thus, the final expression for the inventory volume for linear averaging level control with conventional tuning is

$$V = \frac{1.84(\Delta F_{max})^2}{\left.\frac{dF_{out}}{dt}\right|_{max}} \tag{18.19}$$

EXAMPLE 18.5.

A flow into a vessel has a base value of 2.0 and a maximum step disturbance of 0.20 m³/min. The flow out should have a rate of change that does not exceed 1.0×10^{-3} (m³/min)/min, and the level can vary within ±40% of its middle value. Determine the inventory size to satisfy this requirement when the flow out is manipulated by a PI controller.



Solution. Equation (18.19) can be used directly to calculate the volume to be

$$V = \frac{1.84(0.20\ \text{m}^3/\text{min})^2}{1 \times 10^{-3}\ \text{m}^3/\text{min}^2} = 73.6\ \text{m}^3$$

The area and height can be selected to satisfy this volume (e.g., $A = 36.8$ m^2 and $L = 2$ m). The tuning for this controller can then be calculated for $\Delta L_{\max} = 0.8$ m to be

$$K_c = 0.736\frac{\Delta F_{\max}}{\Delta L_{\max}} = \frac{(0.736)(0.20\ \text{m}^3/\text{min})}{0.8\ \text{m}} = -0.184\frac{\text{m}^3/\text{min}}{\text{m}}$$

$$T_I = \frac{4\xi A}{-K_c} = \frac{4(1)(36.8\ \text{m}^2)}{0.184(\text{m}^3/\text{min})/\text{m}} = 800\ \text{min}$$

The result of this example is a level process and tuning that (just) satisfy the objective on the outlet flow behavior for the specified input step disturbance.

18.7 ▫ IMPLEMENTATION ISSUES

Level control is generally quite straightforward to implement. Many different sensors can be used to determine the inventory in a vessel. The most common is the pressure difference measurement, which is shown in Figure 18.1. Assuming a constant liquid density, the difference in pressure is proportional to the level in the vessel between the two measuring points, called *taps*. Note that the lower tap is usually placed somewhat above the bottom of the vessel, to prevent plugging from a small accumulation of solid contaminants. The level displayed to the operating personnel could be expressed in units of length; however, this would require the people to remember the maximum level in each individual vessel. Therefore, the level is normally displayed as a percentage of the measurement range.

Many other types of level sensors are possible (e.g., Blickley, 1990; Cho, 1982; and Cheremisinoff, 1981). An example is a float that remains at the interface and indicates the level by its physical position as transmitted by a connecting rod. Levels of materials that do not rest evenly in the vessel, such as granular solids, or of very corrosive materials can be measured by sound waves directed at the material from above a vessel. For some accurate measurements, the entire vessel and contents can be weighed.

Level control often uses cascade principles by resetting a flow controller, as shown in Figure 18.1. Usually, this is not to improve the dynamic response to disturbances but to make the operation easier for the operator when the cascade is opened. Level control can be implemented with either linear or nonlinear proportional-only or proportional-integral control algorithms. Both are available as preprogrammed options in most digital controllers.

18.8 ▫ VESSELS IN SERIES

In many chemical plants, units are arranged in series as shown in Figure 18.7. Plants do not usually have many simple tanks in series, but units such as reactors, flash drums, and distillation towers are generally in series and have liquid inventories.

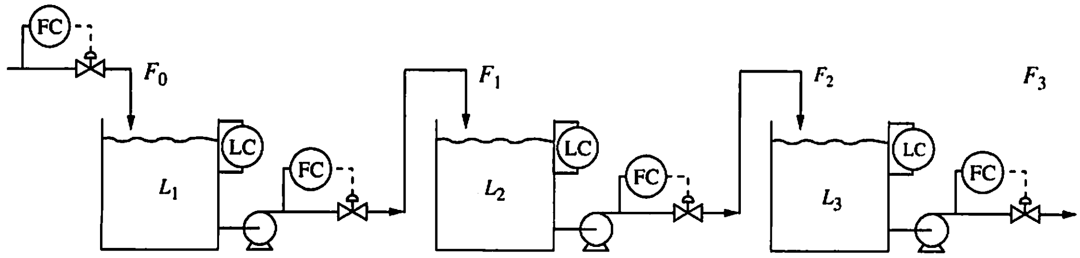


FIGURE 18.7

Design for three levels in series.

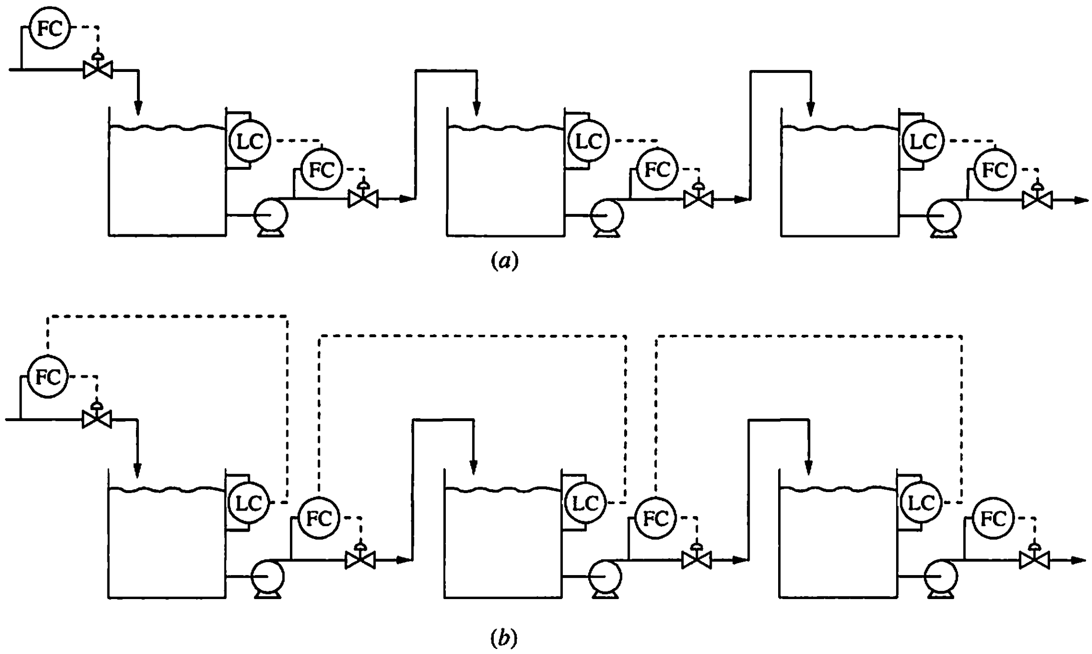


FIGURE 18.8

Two possible control designs for levels in series.

The behavior of these systems is investigated here by considering the simpler, but representative, system of tanks. We will consider two important questions:

1. How can the throughput and levels be controlled?
2. How does a series of levels respond dynamically?

We can answer the first question by analyzing the degrees of freedom in the system. For simplicity, proportional-only controllers are considered, but the results are equally valid for other controller algorithms. The system in Figure 18.8 can be modelled according to the following equations:

For each level ($n = 1$ to 3):

$$A\frac{dL'_n}{dt} = F'_{n-1} - F'_n \qquad (18.20)$$

and *one* of either of these controller equations for each level:

$$F'_n = -K_c L'_n \quad \text{or} \quad F'_{n-1} = -K_c L'_n \tag{18.21}$$

Note that there are six equations and seven variables (three levels and four flows). Thus, one flow rate can be set independently. This result should not be surprising, since level control requires the inlet and outlet flows to be equal at steady state.

Another question to answer is which flow should be set to determine the flow rate. The degrees-of-freedom analysis cannot provide further insight, because any flow is acceptable; thus, this detailed design decision requires more information on the control objectives and process equipment. If no constraints are encountered in the plant, the inlet or feed rate is often set independently, as shown in Figure 18.8*a*. If the production rate should be held constant, the outlet flow is set independently, as shown in Figure 18.8*b*. If an intermediate flow should be constant, as is the case if a constraint like pump capacity or heat exchanger duty is encountered in an intermediate unit, the intermediate flow can be set independently. An interesting control strategy that controls all levels and maximizes the flow rate is given by Shinskey (1981).

Now that the control structure has been determined, the second question about dynamic response can be addressed (Cheung and Luyben, 1979). Based on equation (18.14), the series of three identical level systems shown in Figure 18.8*a* can be combined in the following overall transfer function:

$$\frac{F_3(s)}{F_0(s)} = \left(\frac{T_I s + 1}{\tau^2 s^2 + 2\tau\xi s + 1}\right)^3 \tag{18.22}$$

Since the poles of the individual level control systems are the poles of the series system, if each individual system is overdamped, the overall system is overdamped. However, if the systems are underdamped, the overall system will be underdamped. Dynamic responses of the manipulated flows are given in Figure 18.9*a* through *c* for the system with different damping coefficients in response to a step change in the inlet flow F_0.

The flow adjustments are monotonic for the proportional-only controllers, but the adjustments result in overshoot for all proportional-integral controllers, even those that are critically (or over) damped.

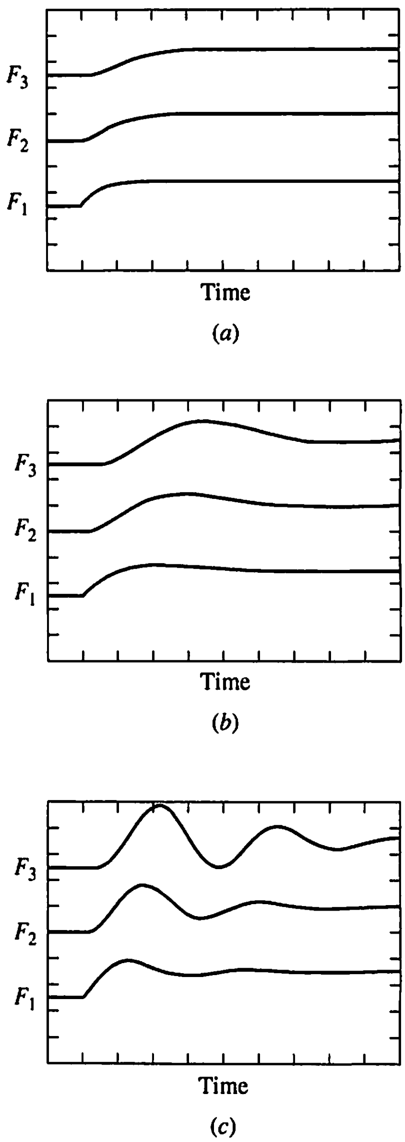


FIGURE 18.9

Response to step of input flow of three series level controllers: (*a*) P-only; (*b*) PI (individual ξ's = 1.0); (*c*) PI level (individual ξ's = 0.5).

> It is important to note that for a step response (1) the manipulated flow for PI control always overshoots its final value and (2) the magnitude of the oscillations *increases* in series systems when each element in the series is underdamped!

A relatively small oscillation at the first level can be magnified, leading to very poor performance, by other downstream levels in the series. Thus, a series process structure of inventories heightens the importance of careful algorithm selection and tuning for each level controller.

18.9 ▨ CONCLUSIONS

The key features of inventory control are the range of control objectives and the need to match the control algorithm with the relevant objective. Feedback control provides excellent tight level control performance, because the system has little or no dead time. Proportional-only or proportional-integral control with simple tuning guidelines is adequate for tight level control.

Analysis of plant requirements indicates that averaging control is appropriate for many level systems. The linear P-only and PI algorithms can achieve averaging control with proper tuning. Improved averaging control can be achieved using a nonlinear PI algorithm when most flow disturbances are of the magnitude and frequency to allow moderate flow manipulations and have the level remain within an acceptable range. This modification is especially advantageous when the system experiences high-frequency disturbances. One should never lose sight of the fact that the performance of averaging level control improves with a large vessel inventory, which must be provided when the process is being designed.

We can derive analytical expressions for the time-domain behavior of level processes and can determine proper tuning rules to achieve specified behavior based on these expressions. The approach used for levels would have been valuable for all feedback systems because of its excellent specification of closed-loop performance. Unfortunately, the approach would not be successful for more complex processes, for which analytical models for closed-loop response cannot be developed. Thus, this excellent approach is limited to a few simple processes.

Smooth overall operation often requires that all flows in the series system have little oscillation. We have seen how levels in series can potentially increase oscillations and have derived models for predicting the responses. These results demonstrate the importance of ensuring that level systems not have small damping coefficients.

Since controlling flows and inventories is an essential aspect of designing controls for multiple units, the material covered in this chapter provides an essential foundation for the control design topics in Part VI.

REFERENCES

Blickley, G., "Level Measured Many Ways," *Cont. Eng.*, *37*, 35–44 (August 1990).

Cheung, T. F., and W. Luyben, "Liquid-Level Control in Single Tanks and Cascades of Tanks with Proportional-Only and Proportional-Integral Feedback Controllers," *IEC Fund.*, *18*, *1*, 15–21 (1979).

Shinskey, F., *Controlling Multivariable Processes*, Instrument Society of America, Research Triangle Park, NC, 1981.

Shunta, J., and W. Feherari, "Non-Linear Control of Liquid Level," *Instr. Techn.*, 43–48 (January 1976).

ADDITIONAL RESOURCES

In addition to the general references cited in Chapter 1, the following books provide specialized information on level sensors and control.

Cheremisinoff, N., *Process Level Instrumentation and Control*, Marcel Dekker, New York, 1981.

Cho, C., *Measurement and Control of Liquid Level*, Instrument Society of America, Research Triangle Park, NC, 1982.

Many other linear and nonlinear controllers similar in purpose to the algorithm presented in Section 18.4 are in use. For a review of the performance of several, see

Cheung, T. F., and W. Luyben, "Nonlinear and Non-Conventional Liquid Level Controllers," *IEC Fund., 19,* 93–98 (1980).

Many other approaches to level control have been proposed. An interesting method that derives an averaging level control algorithm to minimize the maximum rate of change of the manipulated flow is given in

MacDonald, K., T. McAvoy, and A. Tits, "Optimal Averaging Level Control," *AIChE J., 32,* 75–86 (1986).

An alternative to the nonlinear PI algorithm using signal selects (see Chapter 22) has been suggested in

Buckley, P., "Recent Advances in Averaging Level Control," in *Productivity through Control Technology,* April 18–21, 1983, Houston, ISA Paper no. 0-87664-783-2/83/075-11.

The control structure for flows and levels in a system with recycle is shown in Figure Q18.13 and discussed in

Buckley, P., "Material Balance Control of Recycle Systems," *Instr. Techn.,* 29–34 (May 1974).

The sizing of many inventories in a complex plant is discussed in

Hiester, A., S. Melsheimer, and E. Vogel, "Optimum Size and Location of Surge Capacity in Continuous Chemical Processes," *AIChE Annual Meet.,* Nov. 15–20, 1987, paper 86c.

Model predictive control methods are introduced in the next chapter. Additional approaches to level control using model predictive control (see Chapter 19) are given in

Campo, P., and M. Morari, "Model Predictive Optimal Averaging Level Control," *AIChE J., 35, 4,* 579–591 (1989).
Cutler, C., "Dynamic Matrix Control of Imbalanced Systems," *ISA Trans., 21,* 1–6 (1982).

Level control gives the engineer opportunity to match key closed-loop performance measures to the analytical solution to the transient response. This approach enables the engineer to tailor the performance to a wide range of control objectives.

QUESTIONS

18.1. Two tanks in series are placed upstream of a chemical reactor that is sensitive to feed concentration disturbances. Each tank has a holdup of 19 m^3, which is controlled approximately constant, and the design feed rate is 2 m^3/min. If the concentration of the inlet to the first tank has a concentration variation that can be approximated as $20 \sin(1.05t)$, what is the variation in the feed concentration to the reactor?

18.2. Two tanks are placed in series to attenuate flow rate disturbances. Each has a holdup time of τ_H minutes and is controlled by a linear PI controller. If the inlet flow variation is $A \sin(\omega t)$, what is the minimum variation in the flow rate leaving the second tank?

18.3. It was stated that the controller algorithm introduced in Section 18.4 is nonlinear. Using the definition of linearity (see Section 3.4), prove that the algorithm is nonlinear.

18.4. (*a*) Demonstrate that a proportional-only controller for a single level with a holdup time of 5 min and no instrumentation dynamics can have an arbitrarily large controller gain and remain stable.

(*b*) If the system in (*a*) has sensor dynamics of a first-order system with a time constant of 10 sec and valve dynamics of a first-order system with a time constant of 3 sec, what is the ultimate gain of the proportional-only controller? What would be a good choice for the controller gain?

18.5. Averaging level control implements relatively detuned feedback control. Since the integral mode is the "slow" mode, it might seem as though it should be used for control. To investigate why level controllers are predominantly proportional controllers, carry out the following development. Derive the transfer function for a level process under integral-only feedback control. Determine the dynamic response of the level for a step change in the uncontrolled flow. Is this good control performance?

18.6. The derivative mode does not seem to be used in level control. State whether you agree with this decision and why.

18.7. For each of the systems in Figure Q18.7, the flow in (F_{in}) can change independently of the inventory in the vessel. Each is described briefly:

(*a*) A heat exchanger in which the liquid in the vessel boils and the duty is proportional to the heat transfer area

(*b*) An open tank containing a liquid with a constant flow out

(*c*) A gas-filled system with a moving roof and a constant mass on the roof; the gas exits through a partially open restriction

(*d*) A gas-filled system with constant volume; the gas exits through a partially open restriction

(i) For all systems without feedback control ($K_c = 0$), assume that the material balance was initially at steady state, and derive the response to a step change in the inlet flow rate. Is each system self-regulatory or not?

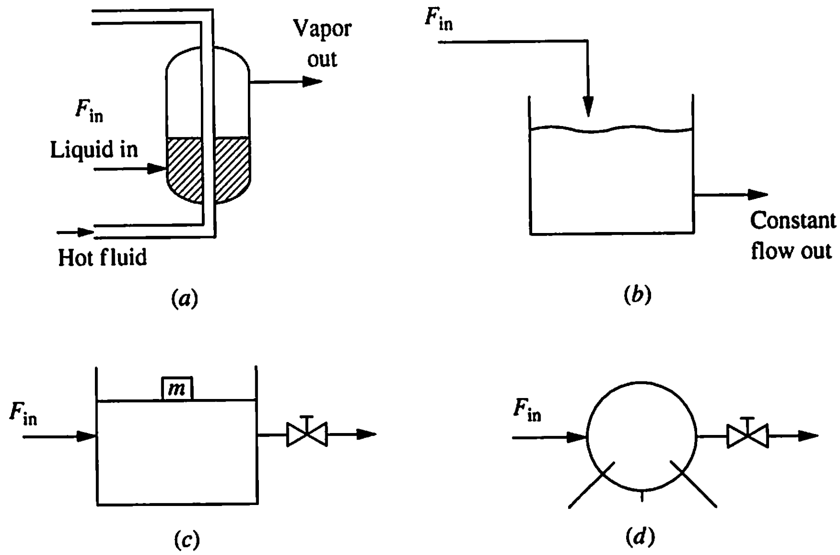


FIGURE Q18.7

(ii) Determine the proper variable to measure to determine the inventory in each system, and describe how it should be controlled, i.e., what should be manipulated?

18.8. The closed-loop dynamic responses for the manipulated flow of a level process under PI control experience overshoot of their final steady-state values in response to a step in flow disturbance.

(*a*) Describe why this occurs and determine steps to prevent this overshoot.
(*b*) In Chapter 5, criteria were derived for transfer function's numerator zero that would lead to an overshoot of the output in response to an input step change. Verify that the criteria are met for $\xi = 1$.

18.9. The value of the small controller gain in the nonlinear level control was recommended to be about 1/20 of the large gain. Describe the performance of the nonlinear level control system with $K_{cS} = 0$ to

(*a*) A large step change in the uncontrolled flow
(*b*) A sine of *small* amplitude in the uncontrolled flow
(*c*) Based on these results, would you support the general recommendation of a zero value for the small controller gain? Under what special circumstances would this be advisable?

18.10. In Section 18.7, control of levels in series was discussed. Sketch on Figure 18.7 the control design when the flow leaving the second vessel is set (constant) by flow control.

18.11. Feedforward control was not considered in this chapter. Discuss whether feedforward control would improve (1) tight level control and (2) averaging level control.

18.12. The system of vessels in series (e.g., Figure 18.7*a*) experiences periodic changes to the operating conditions of upstream units, during which the feed composition from upstream units changes substantially. The amount of mixed material produced during these infrequent and planned changes is to be minimized. What steps would you suggest to minimize the mixing without changing the equipment given in the figure?

18.13. The system of units with a recycle solvent stream is shown in Figure Q18.13. Solvent is added to the main process stream before the stirred-tank reactor and is separated in the flash drum. The solvent is collected, purified in the fixed-bed chemical reactor, and stored. The solvent is heated prior to being mixed with the feed. The feed flow rate is determined elsewhere and can be considered uncontrollable for this question. Also, the maximum purge and makeup flows are 1/10 of the normal solvent flow rate, and the material sent to purge cannot be recycled to the process.

(*a*) Design a control system that (1) ensures solvent addition at the desired ratio in the feed flow and (2) maintains all inventories in acceptable ranges. You may add sensors but make no other changes to the process equipment.

(*b*) Discuss the data and computations required to determine the size of the tanks, especially the middle solvent storage tank.

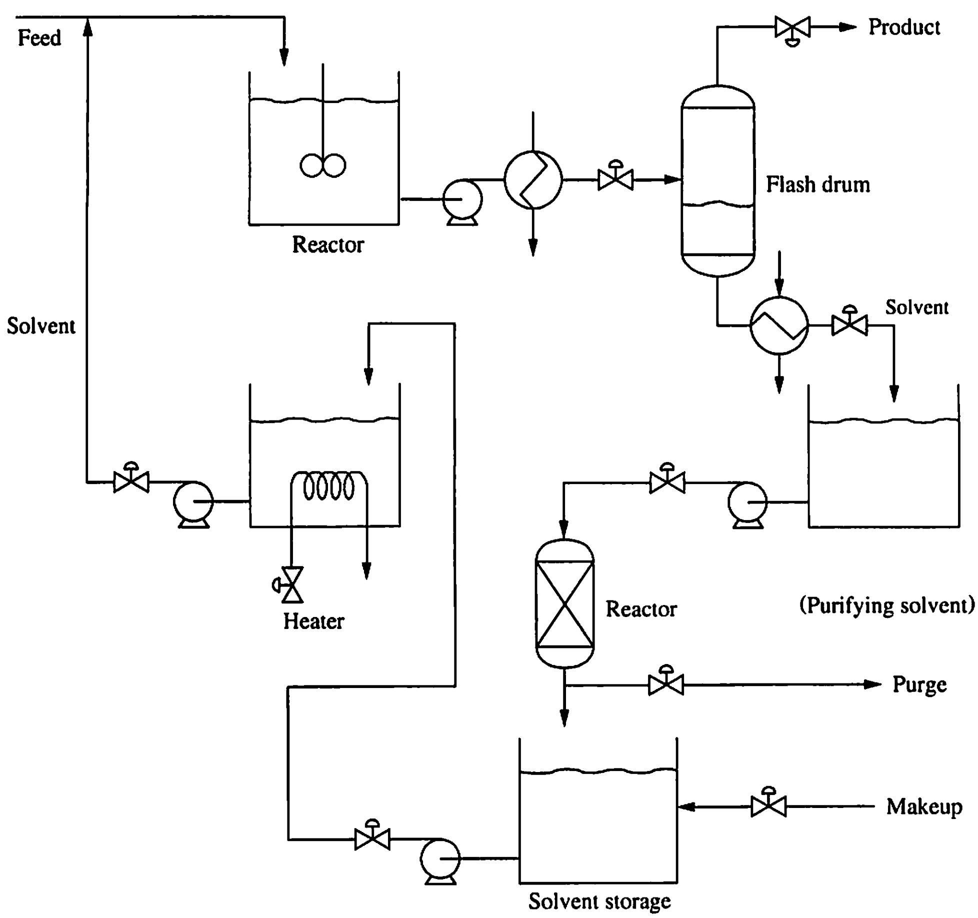


FIGURE Q18.13

(*c*) Discuss how to determine the proper flow rates for the purge and makeup flows. Could they both properly be nonzero concurrently?

18.14. Level controller tuning was not based on the methods and guidelines developed in Chapters 9 and 10. Why?

18.15. Verify the derivation of equations (18.8), (18.9), and (18.11) for the closed-loop response to a step disturbance for a level under PI control.

18.16. For both averaging and tight level control, sketch three examples of processes that should have this type of control and explain why.

18.17. Proposed steps for digital implementation of the nonlinear proportional-integral controller are given below. Discuss whether this implementation satisfies the algorithm described in the chapter, and if not, prescribe modifications.

(1) Read measurement L_n and operator entry L_{SPn}.
(2) Retrieve parameters K_{cS}, K_{cL}, L'_B, and T_I.
(3) Retrieve stored value; S^*.
(4) Set $K_c = K_{cL}$.
(5) If $|L_{SP} - L_n| < L'_B$, then set $K_c = K_{cS}$.
(6) Set $MV_n = K_c\{(L_{SPn} - L_n) + 1/T_I[S^* + \Delta t(L_{SPn} - L_n)]\}$.
(7) Store L_n and $\sum_{i=0}^{n}(\Delta t)(L_{SPi} - L_n) = S^*$.
(8) Wait Δt, then go to step 1.

18.18. Develop a method for determining the size of an inventory for averaging control based on the response of the system to a sine flow rate disturbance using frequency response principles.