Performance of Feedback Control Systems

CHAPTER 13

13.1 INTRODUCTION

As we have learned, feedback control has some very good features and can be applied to many processes using control algorithms like the PID controller. We certainly anticipate that a process with feedback control will perform better than one without feedback control, but how well do feedback systems perform? There are both theoretical and practical reasons for investigating control performance at this point in the book. First, engineers should be able to predict the performance of control systems to ensure that all essential objectives, especially safety but also product quality and profitability, are satisfied. Second, performance estimates can be used to evaluate potential investments associated with control. Only those control strategies or process changes that provide sufficient benefits beyond their costs, as predicted by quantitative calculations, should be implemented. Third, an engineer should have a clear understanding of how key aspects of process design and control algorithms contribute to good (or poor) performance. This understanding will be helpful in designing process equipment, selecting operating conditions, and choosing control algorithms. Finally, after understanding the strengths and weaknesses of feedback control, it will be possible to enhance the control approaches introduced to this point in the book to achieve even better performance. In fact, Part IV of this book presents enhancements that overcome some of the limitations covered in this chapter.

Two quantitative methods for evaluating closed-loop control performance are presented in this chapter. The first is frequency response, which determines the response of important variables in the control system to sine forcing of either the disturbance or the set point. Frequency response is particularly effective in determining and displaying the influence of the frequency of an input variable on control performance. The second quantitative method is simulation, involving numerical solution of the equations defining all elements in the system. This method is effective in giving the entire transient response for important changes in the forcing functions, which can be any general function. Both of these methods require computations that are easily defined but very time-consuming to perform by hand. Fortunately, the calculations can be programmed using simple concepts and executed in a short time using digital computers.

After the two methods have been explained and demonstrated, they are employed to develop further understanding of the factors influencing control performance. First, a useful performance bound is provided that defines the best performance possible through feedback control. Then, important effects of elements in the feedback system are analyzed. In one section the effects of feedback and disturbance dynamics on performance are clarified. In another section the effects of control elements, both physical equipment and algorithms, on control performance are evaluated. The chapter concludes with a table that summarizes the salient effects of control loop elements on control performance.

13.2 □ CONTROL PERFORMANCE

Many measures of control performance are possible, and each is appropriate in particular circumstances. The important measures are listed here, and the reader is referred to Chapter 7 to review their meanings.

- Integral error (IAE, ISE, etc.)
- Maximum deviation of controlled variable
- Maximum overshoot of manipulated variable
- Decay ratio
- Rise time
- Settling time
- Standard deviation of controlled and manipulated variables
- Magnitude of the controlled variable in response to a sine disturbance

Two additional factors should be achieved for control performance to be acceptable; generally, they are not difficult to achieve but are included here for completeness of presentation. The first is zero steady-state offset of the controlled variable from the set point for steplike input changes. For nearly all control systems, zero offset is a desirable feature, and control systems must use a controller with an integral mode to achieve this objective. An important exception where zero offset is not required occurs with some level controllers. Level control is addressed in Chapter 18, where different control performance criteria from those used in this chapter are introduced.

The second factor is stability. Clearly, we want every control system to be stable; therefore, control algorithms and tuning constants are selected to give stable performance over a range of operating conditions. It is very important to recognize that stability places a limit on the maximum controller gain and, in a sense, the control system performance. Without this limit, proportional-only controllers with very high gains might provide tight control of the controlled variable in many applications.

In this chapter we will confine our discussion to control systems that require zero offset and to controller tuning constant values that provide good performance over a reasonable range of operating conditions.

Also, we recognize that no general boundary exists between good and poor process performance. A maximum controlled-variable deviation of 5°C may be totally unacceptable in one case and result in essentially no detriment to operation in another case. In this chapter we identify the key factors influencing control performance and develop quantitative methods for predicting performance measures that can be applied to a wide range of processes; the desired value or limit for each measure will depend on the particular process being considered. In evaluating control performance, we will use the following definition.

Control performance is the ability of a control system to achieve the desired dynamic responses, as indicated by the control performance measures, over an expected range of operating conditions.

This definition of performance includes both set point changes and disturbances. The phrase "over an expected range of operating conditions" refers to the fact that we never have perfect information on the process dynamics or disturbances. Differences between model and plant are inevitable, whether the models were derived analytically from first principles or were developed from empirical data such as the process reaction curve. In addition, differences occur because the plant dynamics change with process operating conditions (e.g., feed flow rate and catalyst activity). Since any model we use has some error, the control system must function "well" over an expected range of errors between the real plant and our expectation, or model, of the plant. The expected range of conditions can be estimated from our knowledge of the manner in which the plant is being operated (values of feed flow, reactor conversion, and so forth).

The ability of a control system to function as the plant dynamics change is sometimes referred to as *robust* control. However, throughout this book we will consider performance to include this factor implicitly without expressly including the word *robust* every time. To reiterate, *we must always consider our lack of perfect models and changing process dynamics when analyzing control performance.*

It is important to emphasize that the performance of a control system depends on all elements of the system: the process, the sensor, the final element, and the controller. Thus, all elements are included in the quantitative methods described in the next two sections, and important effects of these elements on performance are explored further in subsequent sections.

13.3 ◻ CONTROL PERFORMANCE VIA CLOSED-LOOP FREQUENCY RESPONSE

Continuously operating plants experience frequent, essentially continuous, disturbances, so predicting the control system performance for this situation is very important. The approach introduced here is very general and can be applied to any linear plant, not just first-order-with-dead-time, and any linear control algorithm. Also, it provides great insight into the influence of the frequency of the input (set point and disturbance) changes on the effectiveness of feedback control.

The approach is based on the frequency response methods introduced in previous chapters. Frequency response calculates the system output in response to a sine input; we will use this approach in evaluating control system performance by assuming that the input variable—set point change or disturbance—is a sine function. While this is never exactly true, often the disturbance is periodic and behaves approximately like a sine. Also, a more complex disturbance can often be well represented by a combination of sines (e.g., Kraniauskas, 1992); thus, frequency response gives insight into how various frequency components in a more complex input affect performance.

The control performance measure in this section is the amplitude ratio of the controlled variable, which can be considered the deviation from set point because the transfer function equations are in deviation variables. The frequency response of a stable, linear control system can be calculated by replacing the Laplace variable s with $j\omega$ in its transfer function. The resulting expressions describe the amplitude ratio and phase angle of the controlled variable after a long enough time that the nonperiodic contribution to the solution is negligible. The control system in Figure 13.1 is the basis for the analysis, and this system has the following transfer function in response to a disturbance:

$$\frac{\mathrm{CV}(s)}{D(s)} = \frac{G_d(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)} \tag{13.1}$$

It is helpful to consider the amplitude ratio of the controlled variable to the disturbance in equation (13.1), which can be expressed as the product of two factors:

$$\frac{|\mathrm{CV}(j\omega)|}{|D(j\omega)|} = \left[|G_d(j\omega)| \left| \frac{1}{1 + G_p(j\omega)G_v(j\omega)G_c(j\omega)G_s(j\omega)} \right| \right] \tag{13.2}$$

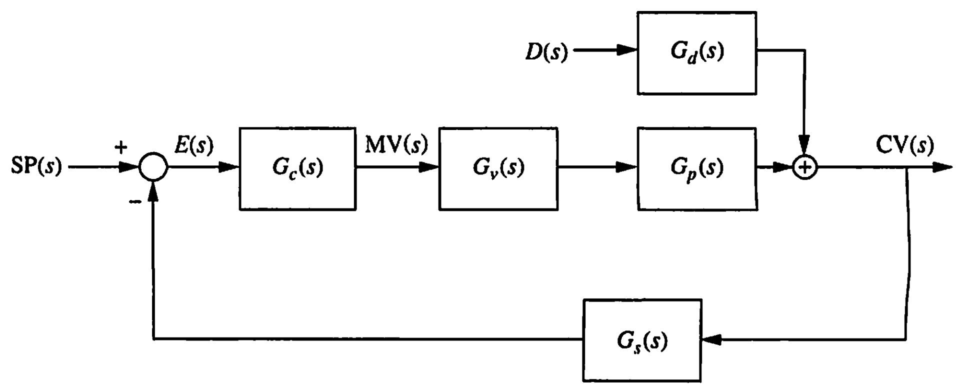


FIGURE 13.1

Block diagram of feedback control system.

The first factor of the amplitude ratio is the numerator, which contains the open-loop process disturbance model. The second factor is the contribution from the feedback control system. The frequency responses of the factors are given in Figure 13.2*a* and *b* and are referred to in analyzing the frequency response of the closed-loop system. The results in Figure 13.2 are for the (arbitrary) system

$$G_p(s)G_v(s)G_s(s) = \frac{1.0e^{-15s}}{20s+1} \qquad G_c = 0.60\left(1+\frac{1}{30s}\right) \qquad G_d = \frac{0.48}{20s+1}$$

When interpreting these plots, it is helpful to remember that (unachievable) perfect control would result in no controlled-variable deviation for all frequencies; in other words, the output (CV) amplitude would be zero for all frequencies. The closed-loop system is first considered at limits of very low and very high frequency. This analysis makes use of equation (13.2) and Figure 13.2*a* and *b*. For disturbances with a very low frequency, the first factor (i.e., the process through which the disturbance travels) does not attenuate the disturbance; thus, its magnitude is large. (The disturbance dynamics are assumed similar to the feedback dynamics for this example.) However, the relatively fast feedback control loop will effectively attenuate a disturbance in this frequency range; thus, the magnitude of the feedback factor is small. The control system response is the product of the two magnitudes; therefore, the control system provides good performance at input frequencies much lower than the critical frequency, because of feedback control. Note that the integral mode of the PI controller is especially effective in rejecting slow disturbances and that in general, feedback control systems provide good control performance at very low disturbance frequencies.

For disturbances at the other extreme of very high frequency, the feedback controller is not effective, because the disturbance is faster than the control loop can respond. In this case the magnitude of the second factor is nearly 1. However,

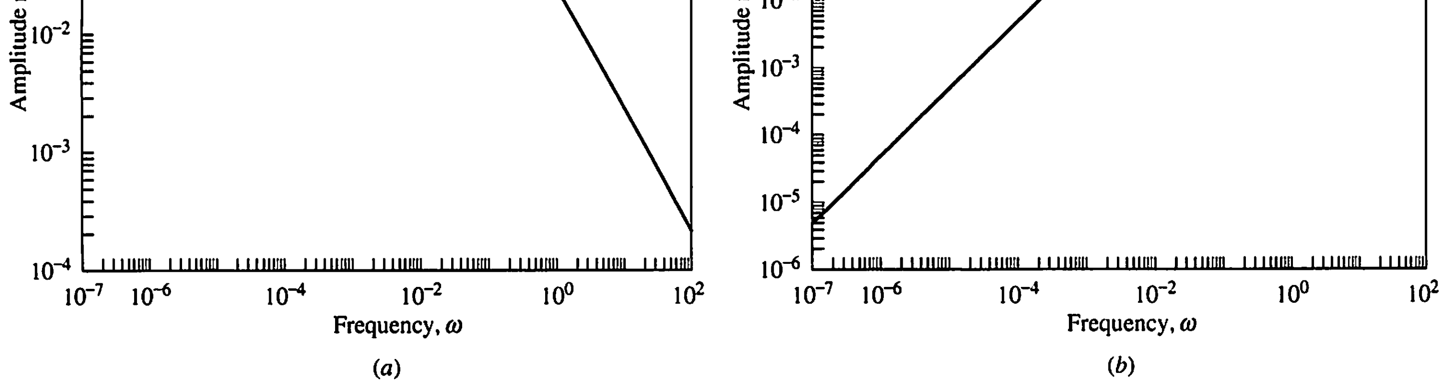


FIGURE 13.2

Amplitude ratios in equation (13.2): (*a*) numerator; (*b*) denominator.

the disturbance process, as long as it consists of first- or higher-order time constants (and not simply gains and dead times), filters the high-frequency disturbance. This filter results in a small magnitude of $|G_d(j\omega)|$, reducing the magnitude of the controlled variable substantially. Therefore, the feedback control system provides good control performance for very high frequencies as well. Note that the good performance is not due to feedback control but rather to the disturbance time constant(s), which in this range is much larger than the disturbance period (i.e., $1/\tau_d \ll \omega_d$).

For intermediate frequencies, a harmonic or resonant peak occurs. This peak represents the most difficult frequencies for the feedback control system. In fact, for some systems the control system can perform *worse* than the same plant without control, indicating that disturbances can be slightly amplified by the feedback control loop around the harmonic frequency.

The general shape of the closed-loop frequency response to a disturbance for most feedback controller systems is similar to the curve in Figure 13.3. It is important that the engineer understand the reasons for the behavior in the low-, intermediate-, and high-frequency regions. Many disturbances in process plants have low frequencies, because they result from the changing operation of slowly responding systems such as the composition of flows from large upstream feed tanks. Many very fast disturbances occur due to imperfect mixing and high-frequency pressure disturbances. For both disturbances, feedback control performance tends to be good. However, many disturbances also occur around the critical frequency

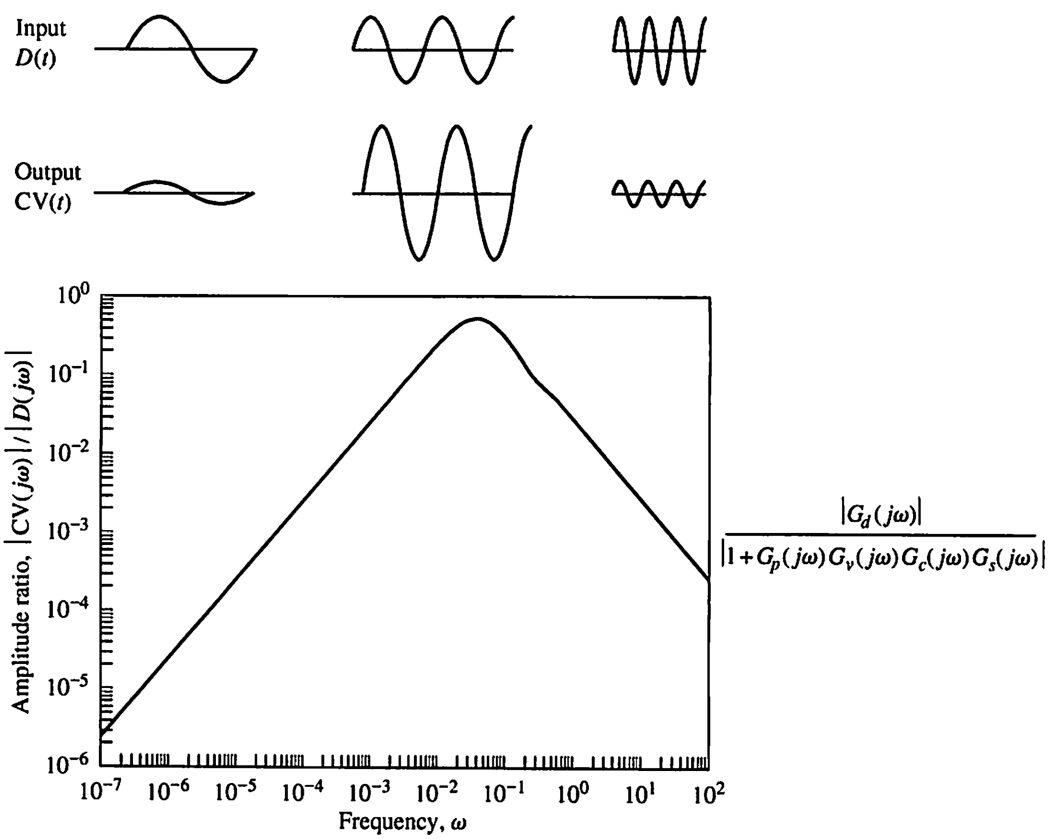


FIGURE 13.3

Frequency response of feedback-controlled variable to disturbance.

of a feedback loop, because oscillations caused by an integrated process under feedback control tend to be in the same frequency range.

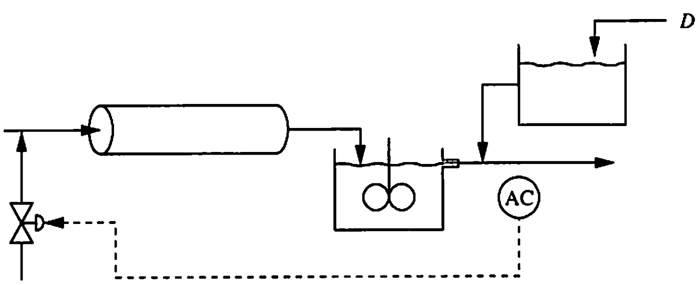
> **Disturbances around the closed-loop resonant frequency are essentially uncontrollable with *any single-loop feedback controller*, and therefore such disturbances should be prevented by changes to the process design or attenuated using enhancements discussed in Part IV.**

EXAMPLE 13.1.

The plants presented in Figure 13.4 are subject to periodic disturbances. All plants have the same equipment structure, but they have different equipment sizes. They can all be modelled as first-order-with-dead-time processes, and the dynamics of the sensor and valve are negligible. Determine the control performance in response to a disturbance (D) possible with the four designs and rank them according to the amplitude ratios achieved by PI controllers.

The solution to the example involves calculating the closed-loop frequency response for each case. The calculations are based on equation (13.2), with the appropriate transfer functions for the individual elements—in this case, a first-order-with-dead-time process, a first-order disturbance, and a PI controller. The calculation of the amplitude ratio follows the same procedure used in Chapter 10, where s is replaced by $j\omega$ in the transfer function; then the magnitude of the complex expression is determined. The results of the algebraic manipulations for this example are given in equation (13.3); recall that the frequency response could also be evaluated using computer methods not requiring these extensive algebraic manipulations.

$$\text{Amplitude ratio} = |G_d(j\omega)| \left| \frac{1}{1 + G_c(j\omega)G_p(j\omega)} \right| \tag{13.3}$$



Case	K_p	θ	τ	τ_d
A	1.0	1.0	1.0	1.0
B	1.0	4.0	4.0	1.0
C	1.0	0.5	1.5	1.0
D	0.1	0.5	1.5	1.0

FIGURE 13.4

Schematic of process with model parameters for Example 13.1.

where $$|G_d(j\omega)| = \frac{K_d}{\sqrt{1+\omega^2\tau_d^2}} \quad \text{with } K_d = 1$$

$$\left|\frac{1}{1+G_c(j\omega)G_p(j\omega)}\right| = \frac{\sqrt{(AC+BD)^2+(BC+AD)^2}}{C^2+D^2}$$

$$A = -T_I\tau_p\omega^2 \qquad B = T_I\omega$$
$$C = K_pK_c[\cos(-\theta\omega) - T_I\omega\sin(-\theta\omega)] - T_I\tau_p\omega^2$$
$$D = K_pK_c[\sin(-\theta\omega) + T_I\omega\cos(-\theta\omega)] + T_I\omega$$

In each case, the PI controller has to be tuned; the tuning for this example is given below based on the Ciancone correlations in Figure 9.9*a* and *b*.

Case	$\theta/(\theta+\tau)$	K_cK_p	$T_I/(\theta+\tau)$	K_c	T_I
A	0.5	0.85	0.75	0.85	1.5
B	0.5	0.85	0.75	0.85	6.0
C	0.25	1.70	0.65	1.70	1.3
D	0.25	1.70	0.65	17.0	1.3

The best control performance has the smallest amplitude ratio (i.e., the smallest deviation from set point). These calculations have been performed, and the results are given in Figure 13.5, which shows that the best performance is possible with designs C and D. The next best is case A, and the worst is case B.

Since the disturbance transfer function is the same for all cases, the processes with the longest dead time and the longest dead time plus time constant in the feedback path are more difficult to control; this explains why case B has the poorest

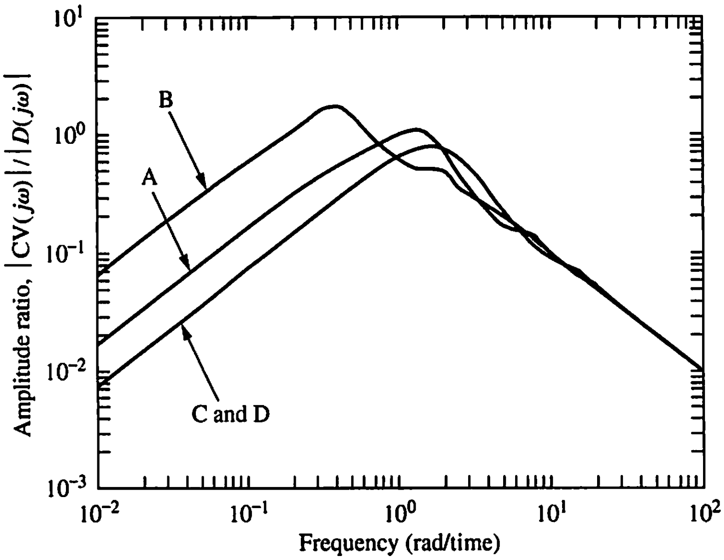


FIGURE 13.5

Closed-loop frequency responses for the cases in Example 13.1.

performance and why case A is not as good as C and D. Note that processes C and D have the same dynamics and differ only in their gains. Thus, the controller gain can be selected to achieve the same K_pK_c and the same control performance. (This result assumes that the manipulated variable can be adjusted over a larger range for the process with the smaller process gain.) In addition to finding the best process, we have identified a region of disturbance frequency for which feedback control will not function well. Process changes or control enhancements would be in order if disturbances with large magnitudes were expected to occur in this frequency range.

EXAMPLE 13.2.

Normal plant disturbances have many causes with different frequencies. This example presents a simple case of two disturbances. As depicted in Figure 13.6, the input disturbance is the sum of two sine waves that have the same phase and have the amplitudes and frequencies given in the following table. The input disturbances are not measured, but sample open-loop dynamic data of the output variable [i.e., $G_d(s)D(s)$] are given in Figure 13.7*a*. What is the magnitude of the sine wave of the controlled variable when PI feedback control is implemented for the same disturbance?

	Input No. 1	Input No. 2
Frequency (rad/min)	0.010	0.20
Amplitude	1.0	0.50

The first step in the solution is to calculate the closed-loop frequency response for this process with PI control. The process is first-order-with-dead-time, and the calculations employ equation (13.3) with the following parameters:

$$K_p = 1.0 \quad \tau = 2.0 \quad \theta = 1.0 \quad G_d(s) = 1$$
$$K_c = 1.0 \quad T_I = 2.0$$

The amplitude ratio of each input considered individually can be determined as

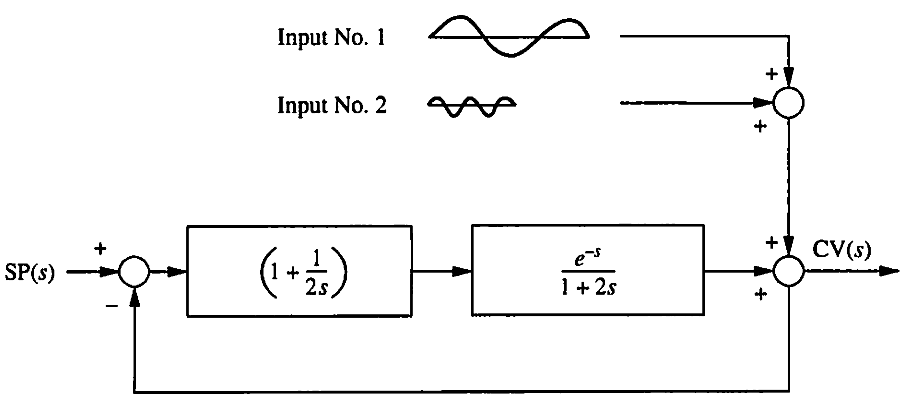
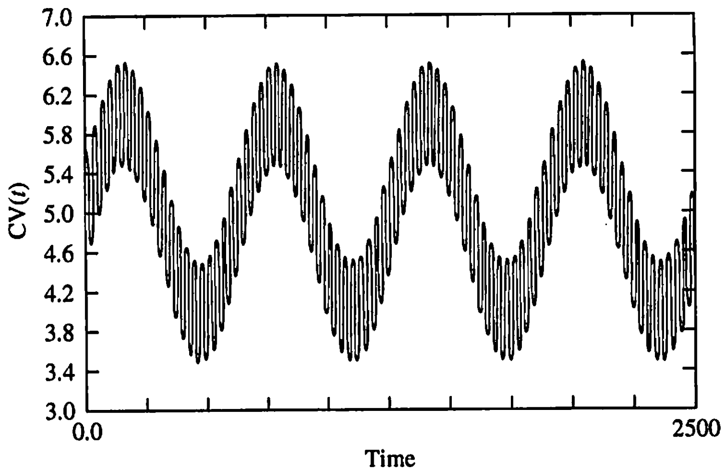
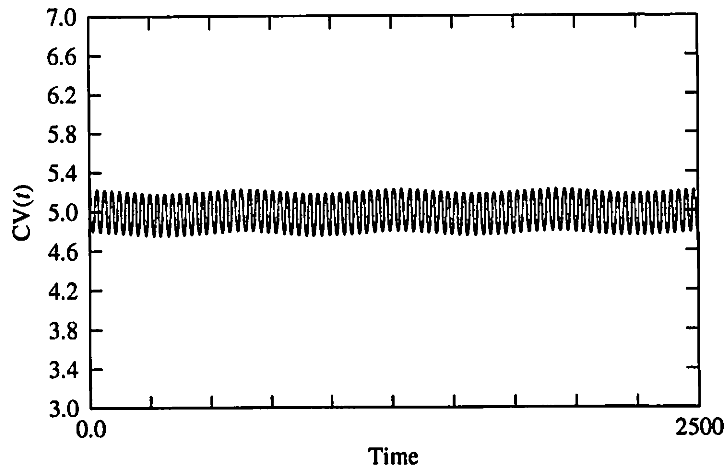


FIGURE 13.6

Schematic showing the system and disturbances considered in Example 13.2.



(*a*)



(*b*)

FIGURE 13.7

Results for Example 13.2: (*a*) disturbance without control; (*b*) closed-loop dynamic response with PI control.

shown in Figure 13.8. The lower-frequency disturbance (input no. 1) has a very small amplitude ratio. Thus, the control performance for this part of the disturbance is good. The amplitude ratio for the higher-frequency input (input no. 2) is not small and is about 0.50, because it is in the region of the resonant frequency. Therefore, input No. 2 contributes most of the deviation for the closed-loop feedback control system.

This analysis can be compared with the dynamic response of the closed-loop control system with the two sine disturbances given in Figure 13.7*b*. The response shows almost no effect of the slow sine disturbance and a significant effect from the faster sine disturbance. The magnitude of the closed-loop simulation, about 0.25, is the same as the prediction from the frequency response analysis, 0.5×0.5. We can conclude from this example that the frequency response method provides valuable insight into which disturbance frequencies will and will not be attenuated significantly by feedback control.

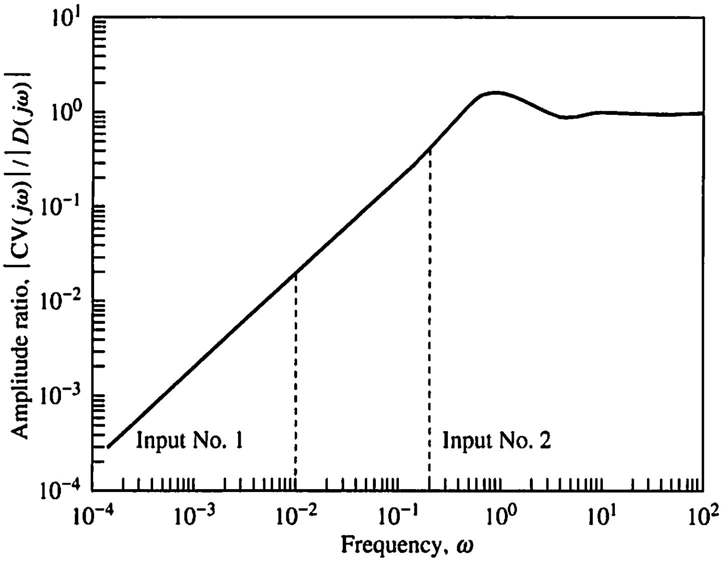


FIGURE 13.8

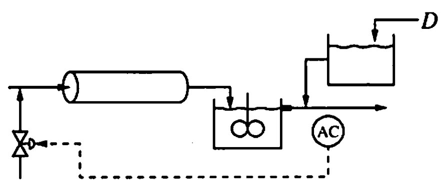
Closed-loop amplitude ratio for Example 13.2.

Most process control systems are primarily for disturbance response, but some have frequent changes to their set points. The frequency response approach developed for disturbance performance analysis can be extended to set point response to determine how well the control system can follow, or *track*, its set point. The following transfer function relates the controlled variable to the set point for the system in Figure 13.1:

$$\frac{\mathrm{CV}(s)}{\mathrm{SP}(s)} = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)} \tag{13.4}$$

The amplitude ratio of this transfer function can be calculated using standard procedures (setting $s = j\omega$) and plotted versus frequency of the set point variation. Perfect control would maintain the controlled variable exactly equal to the set point; in other words, the amplitude ratio would be equal to one (1.0) for all frequencies. Very good control performance is achieved for very low frequencies, when the feedback control system has time to respond to slow set point change. As the frequency increases, the control performance becomes poorer, because the set point variations become too fast for the feedback control system to track closely. Again, a resonant peak can occur at intermediate frequencies.

EXAMPLE 13.3.



Calculate the set point frequency response for the plant in Example 13.1, case C.

The transfer functions of the process and controller are given in Example 13.1. The result of calculating the amplitude ratio of equation (13.4) is given in Figure 13.9. As shown in the figure, the control system would provide good set point tracking (i.e., an amplitude ratio close to 1.0) for a large range of frequencies. The frequency range for which the amplitude ratio responds satisfactorily is often referred to as the *system bandwidth;* taking a typical criterion that the amplitude ratio of 1.0 to 0.707 is acceptable, the bandwidth of this system is frequencies from 0.0 to about 3 rad/time.

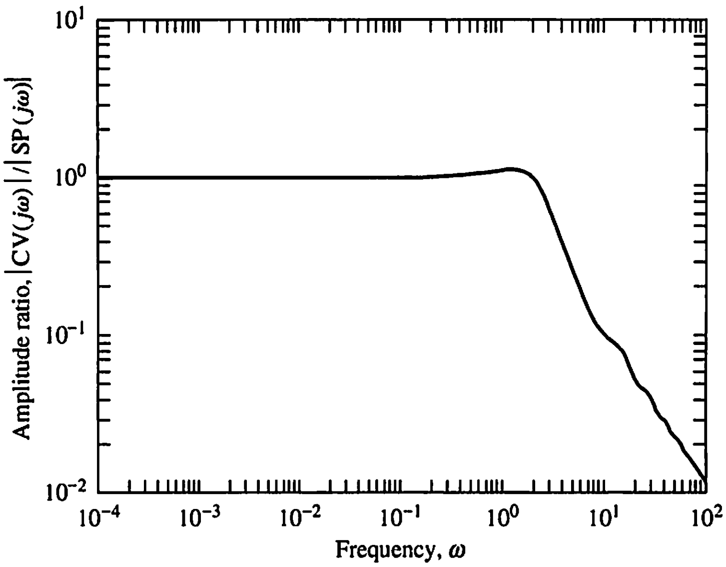


FIGURE 13.9

Closed-loop frequency response for the set point response in Example 13.3.

The calculation of the frequency response for the closed-loop system is performed by applying the same principles as for open-loop systems. However, the calculations are much more complex. The frequency response for closed-loop systems requires that the transfer function be solved for the magnitude, and the results must be derived for each system individually, as was done analytically in equation (13.3). Clearly, this amount of analytical manipulation could inhibit the application of the frequency response technique.

In the past, graphical correlations have been used to facilitate the calculations for a limited number of process and controller structures. The Nichols charts (Edgar and Hougen, 1981) are an example of a graphical correlation approach to calculate the closed-loop from the open-loop frequency response. These charts are not included in this book because closed-loop calculations are not now performed by hand.

Since the advent of inexpensive digital computers, the calculations have been performed with the assistance of digital computer programs. Most higher-level languages (e.g., FORTRAN) provide the option for defining variables as complex and solving for the real and imaginary parts; thus, the computer programming is straightforward, basically programming equation (13.2) with complex variables. An extension to the programming approach is to use one of many software packages that are designed for control system analysis, such as MATLAB™. An example of a simple MATLAB program to calculate the frequency response in Figure 13.9 is given in Table 13.1. For simple models, the approach in Example 13.1 can be used, but computer methods are recommended over algebraic manipulation for closed-loop frequency response calculations.

The frequency response approach presented in this section is a powerful, general method for predicting control system performance. The method can be applied to any stable, linear system for which the input can be characterized by a

TABLE 13.1

Example MATLAB™ program to calculate a closed-loop frequency response

```
%   ** EXAMPLE 13.3 FREQUENCY RESPONSE ***
%   this MATLAB M-file calculates and plots for Example 13.3
%   ********************************************
%   parameters in the linear model
%   ********************************************
kp = 1.0 ; taup = 1.5 ; thetap = 0.5;
kc = 1.7  ;  ti = 1.3;
%   ********************************************
%   simulation parameters
%   ********************************************
wstart = .0001  ;   % the smallest frequency
wend   = 100   ;   %  the highest frequency
wtimes = 800  ;   %  number of points in frequency range
omega = logspace ( log10(wstart), log10(wend), wtimes);
jj = sqrt(-1) ;   %  define the complex variable
%   ********************************************
%   put calculations here
%   ********************************************
for kk = 1:wtimes
    s = jj*omega(kk)           ;
     Gp(kk) = kp * exp (- thetap * s) /( ( taup*s + 1)) ;
     Gc(kk) = kc*(1 + 1/ (ti * s));
     G (kk) = Gc(kk)*Gp(kk)/(1 + Gc(kk)*Gp(kk));
     AR(kk) = abs (G(kk));
end  % for cnt
%   **************************************************
%   plot the results in Bode plot
%   **************************************************
loglog( omega, AR)
axis ([ -4 2 -2 1])
xlabel ('frequency, rad/time ')
ylabel ('amplitude ratio')
```

dominant sine. The calculations of the amplitude ratio for a closed-loop system are usually too complex to be performed by hand but are easily performed via digital computation.

> The great strength of frequency response is that it provides a clear indication of the control performance for an input (disturbance or set point change) at various frequencies.

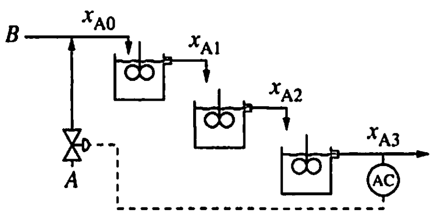
13.4 CONTROL PERFORMANCE VIA CLOSED-LOOP SIMULATION

Solution of the time-domain equations defining the dynamic behavior of the system is another valuable method for evaluating the expected control performance of a design. Unfortunately, the differential and algebraic equations for a realistic control system are usually too complex to solve analytically, although that would be preferred so that analytical performance relationships could be determined. However, numerical solution of the algebraic and differential equations is possible and usually provides an excellent approximation to the behavior of the exact equations.

One reason for using simulation is that control performance specifications are defined in the time domain. The comparison of the predicted performance to the specifications often requires the entire dynamic response—the variables over the entire transient response—to ensure proper dynamic behavior. Thus, the solution to the complete model is required. Also, the engineer likes to see the entire transient response to evaluate all factors, such as maximum deviation, decay ratio, and settling time. The simulation approach is particularly useful in determining the response of a system to a worst-case disturbance. This largest expected disturbance can be introduced, and the resulting response will indicate whether or not all process variables can be maintained within their specified limits.

Numerical methods used to solve ordinary differential equations were described briefly in Chapter 3. Note that equations for all elements in the system—process, instrumentation, and controller—must be solved *simultaneously.* Also, since the solution is numerical, there is no requirement to linearize the equations, although insight from the analysis of linear models is always helpful. Simulation methods have been used to prepare most of the closed-loop dynamic responses in figures for this book.

EXAMPLE 13.4.



Determine the dynamic response of the three-tank mixing process defined in Example 7.2 under PID control to a disturbance in the concentration in stream B of +0.8%.

This is the case considered in Example 9.2, in which the PID tuning was first determined from a process reaction curve. The dynamic response of the closed-loop control system was then determined by solving the algebraic and differential equations describing the system, along with the algorithm for the feedback controller. The following equations summarize the model:

$$\begin{aligned}
E &= SP - x_{A3} \\
v &= K_c\left[E + \frac{1}{T_I}\int_0^t E(t')\,dt' - T_d\frac{dx_{A3}}{dt}\right] + 50 \\
F_A &= 0.0028v \qquad\qquad (13.5) \\
x_{A0} &= \frac{F_B(x_A)_B + F_A(x_A)_A}{F_B + F_A} \\
V_i\frac{dx_{Ai}}{dt} &= (F_A + F_B)(x_{Ai-1} - x_{Ai}) \qquad \text{for } i = 1,3
\end{aligned}$$

The PID controller can be formulated for digital implementation as described in Chapter 11. Also, the differential equations can be solved by many methods; here they are formulated in the discrete manner using the Euler integration method.

Both the process and the controller are executed at the period Δt.

$$E_n = \text{SP}_n - (x_{A3})_n$$

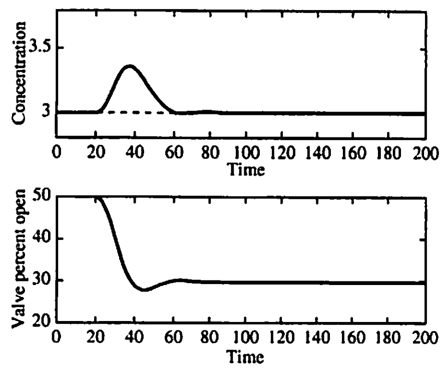
$$(v)_n = (v)_{n-1} + K_c \left\{ E_n - E_{n-1} + \frac{\Delta t E_n}{T_I} + \frac{T_d}{\Delta t}\left[-(x_{A3})_n + 2(x_{A3})_{n-1} - (x_{A_s})_{n-2}\right]\right\}$$

$$(F_A)_n = 0.0028(v)_n \qquad (13.6)$$

$$(x_{A0})_n = \left[\frac{F_B(x_A)_B + F_A(x_A)_A}{F_B + F_A}\right]_n$$

$$(x_{Ai})_{n+1} = (x_{Ai})_n + \frac{\Delta t(F_B + F_A)_n}{V_i}\left[(x_{Ai-1})_n - (x_{Ai})_n\right] \quad \text{for } i = 1, 3$$

The initial conditions are $(x_{Ai})_0 = 3.0\%$ A for $i = 0, 3$ and $(v)_0 = 50\%$ open. The controller tuning constants are $K_c = 30$, $T_I = 11$, and $T_d = 0.8$. The disturbance was a step in $(x_A)_B$ from its initial value of 1.0 to 1.8 at time 20. The execution period was selected to be small relative to the time constants of the process, 0.1 minute. The result of executing the equations (13.6) recursively is the entire transient response. The manipulated and controlled variables are plotted in the adjacent figure. Note that the numerical simulation approach is not limited to linear systems. In fact, this example involves several nonlinearities, e.g., $F_A x_A$.



The simulation method is not restricted to simple input forcing functions, and this flexibility is very useful in estimating likely improvements in control performance. As demonstrated in the previous example, the control performance can be determined based on a model of the feedback process and a model of the disturbance. If the disturbance is a complicated function, a representative sample of the effect of the disturbance on the variable to be controlled can be used as a "model" of the disturbance. The effect of the disturbance(s) can be obtained by collecting *open-loop* data of the variable to be controlled as typical variabilities in plant operation occur.

EXAMPLE 13.5.

PI control is to be applied to the plant with feedback dynamics characterized by a dead time and single time constant. In the plant an undesirable feed component is reacted to a benign effluent component. The outlet concentration is to be controlled by adjusting the feed preheat. The control objective is to maintain the outlet concentration just below its maximum value. Too low a concentration leads to costly side reactions and byproducts; thus, the goal is to reduce the variance. The model, determined by empirical identification, and the controller tuning are as follows:

$$G_p(s)G_v(s)G(s) = \frac{\text{AC}(s)}{v(s)} = \frac{1.0e^{-2s}}{1+2s} \qquad G_c = 1.0\left(1 + \frac{1}{2.5s}\right) \qquad (13.7)$$

A sample of representative dynamic data of the reactor effluent without control is presented in Figure 13.10*a*. Note that some of the variation is of low frequency; feedback control would be expected to be successful in attenuating these low-frequency components. Also, some of the variation is relatively high-frequency, which, we expect, would be difficult to reduce with feedback control.

To predict the performance of the control system, a simulation can be performed using the plant model with the sample disturbance data. This approach

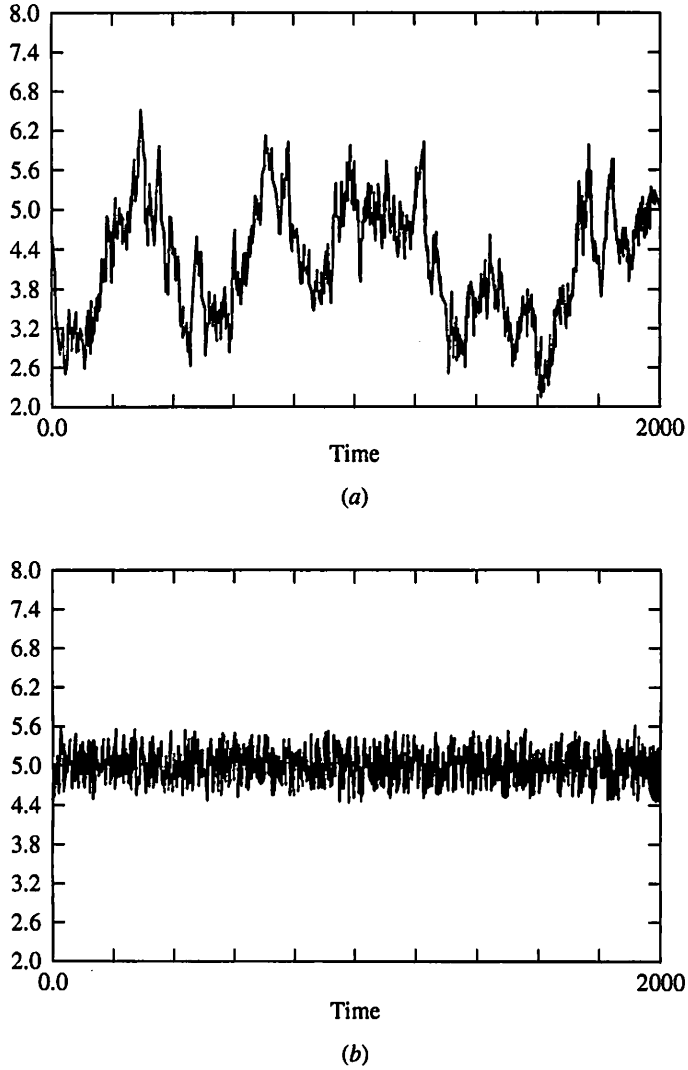


FIGURE 13.10

Reactor outlet concentration, Example 13.5: (*a*) effect of disturbance without control; (*b*) dynamic response with feedback control.

is shown schematically in Figure 13.11, where the digital simulation would introduce the disturbance data collected from the process, Figure 13.10*a*, as the forcing function. Naturally, the controller calculation, here a proportional-integral algorithm, receives the controlled process output, which is the sum of the effects from the manipulated variable and the disturbance. The results of the simulation are given in Figure 13.10*b*. The variability of the controlled variable, measured by standard deviation, has been reduced substantially by feedback control. Analysis of a larger set of data than shown in the figure, which gives a more reliable indication of performance, shows that the standard deviation is reduced by a factor of 5. As expected, the high-frequency components are not substantially reduced by the feedback control system. Because of the smaller variation, the average value of the concentration (i.e., the controller set point) could be changed to realize the benefits from improved control performance.

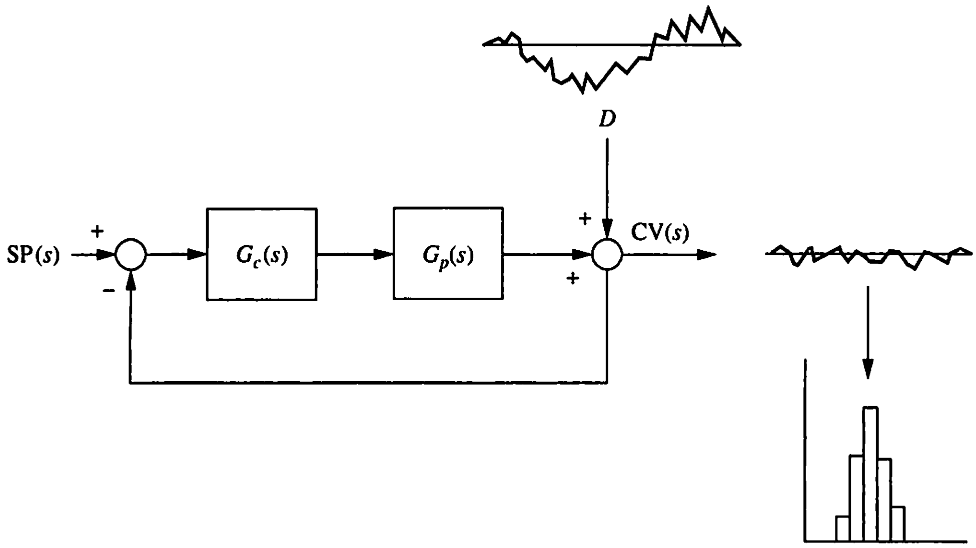


FIGURE 13.11

Schematic of the calculation method for predicting control performance with a complex disturbance model by a simulation method.

This example clearly demonstrates the improvement possible with feedback control and provides a simple, simulation-based method for estimating control performance. The method requires a process model, a controller equation, and a sample of the output variable without control; it provides a prediction of the standard deviation of the manipulated and controlled variables. It can be used in conjunction with the benefits calculations to estimate control benefits quantitatively, as shown in Figure 13.11.

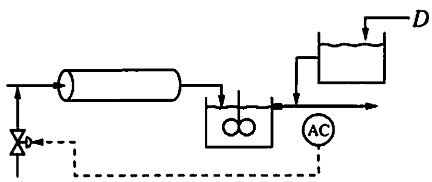
The material in this section has demonstrated that:

> Dynamic simulation via numerical solution of the system equations provides a manner for determining the dynamic performance of a closed-loop process control system. The approach can (1) provide a solution for nonlinear as well as linear systems; (2) consider any input forcing functions; and (3) provide detailed information on all variables throughout the transient response.

Frequency response and dynamic simulation, provide methods required to analyze control systems quantitatively. These methods are applied in the next sections to develop understanding of how specific aspects of process dynamics and the PID controller influence performance.

13.5 □ PROCESS FACTORS INFLUENCING SINGLE-LOOP CONTROL PERFORMANCE

Because the process ($G_p(s)$ and $G_d(s)$), instrumentation ($G_v(s)$ and $G_s(s)$), and the controller ($G_c(s)$) appear in the closed-loop transfer function in equation (13.1), all elements in the feedback system influence its dynamic response and control performance. It is tempting to believe that a cleverly designed controller algorithm can compensate for a difficult process; however, the process imposes limitations on the achievable feedback control performance, *regardless of the feedback algorithm used.* An understanding of the effects of process dynamics on control performance enables us to design plants that are easier to control, recognize limits to the performance of single-loop feedback control, and design enhancements. The next topic establishes a bound on the best achievable feedback control performance that gives valuable insight into the effects of process dynamics.



A Bound on Achievable Performance

The first topic introduced in this section is the performance bound (i.e., the best achievable performance) for a feedback system. The best performance is explained with reference to the process shown in Figure 13.4, where the control system is subjected to a *step change* disturbance. (Note that this concept is applicable to more general processes than Figure 13.4.) The dynamic responses of the controlled and manipulated variables are graphed versus time in Figure 13.12, and several important features of the response are highlighted. First, note that the effect of the feedback adjustment has no influence on the controlled variable for a period of time equal to the dead time in the feedback loop. Therefore, the integral error and maximum deviation shown in Figure 13.12 cannot be reduced lower than the open-loop response for time from zero (when the disturbance first affects the controlled variable) to the dead time. For the special case of a step disturbance with magnitude ΔD and a first-order disturbance transfer function with gain K_d and time constant τ_d, the limiting integral error and maximum deviation can be simply evaluated by the equations

$$E = K_d(1 - e^{(-t/\tau_d)})\Delta D \qquad \text{for } 0 \le t \le \theta \tag{13.8}$$

$$\begin{aligned} \text{IAE}_{\min} &= \int_0^\theta |E|\,dt \\ &= |K_d \Delta D| \int_0^\theta \left|(1 - e^{-(t/\tau_d)})\right| dt \\ &= |K_d \Delta D| \left[\theta + \tau_d\left(e^{-\theta/\tau_d} - 1\right)\right] \end{aligned} \tag{13.9}$$

$$|E_{\max}|_{\min} = |K_d \Delta D| \left(1 - e^{-(\theta/\tau_d)}\right) \tag{13.10}$$

$\text{IAE}_{\min}$ represents the minimum IAE possible, and $|E_{\max}|_{\min}$ represents the minimum value possible for the maximum deviation for a feedback system with dead time θ, a step disturbance, and a disturbance time constant of τ_d. *No single-loop feedback controller* can reduce the values further. As shown in the figure, these values provide a useful bound with which to evaluate control performance. The important conclusion from this discussion is that

> **The dead time in the feedback path is the facet of the process that usually limits the control performance.**

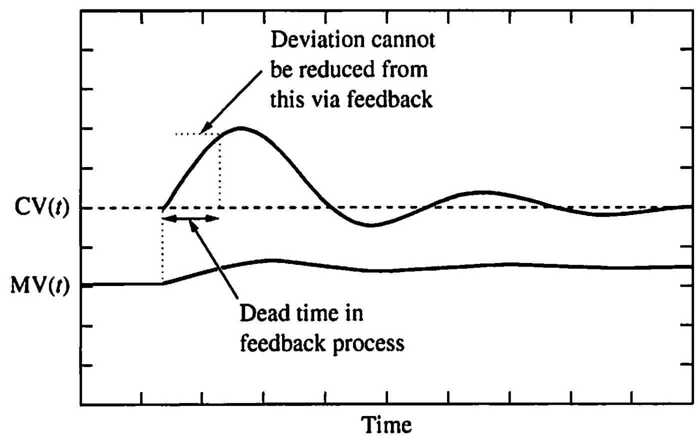


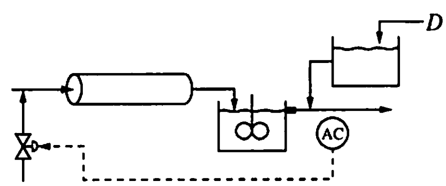
FIGURE 13.12

Typical dynamic response for a feedback control system.

The theoretical best achievable control performance cannot usually be realized with a PID control algorithm, although the PID often provides entirely satisfactory performance. Methods exist for deriving the control algorithms giving the theoretical best or "optimal" control, with *optimal* defined several ways, such as minimum integral of error squared (Newton, Gould, and Kaiser, 1957; Astrom and Wittenmark, 1984). It is important to recognize that these optimal controllers can result in excessive variation in the manipulated variable, and their performance can be very sensitive to model errors. Therefore, the "optimal" algorithms are not often applied in the process industries, although their concepts are useful in determining the achievable performance bounds in equations (13.9) and (13.10).

EXAMPLE 13.6.

The potential designs shown in Figure 13.4, plus one additional, have been proposed for a plant. It is expected that all designs have nearly the same capital cost. The major disturbance is an occasional step with magnitude of 2.5 units. Which of the designs will have the best control performance? The dynamic model parameters are summarized in the following table.

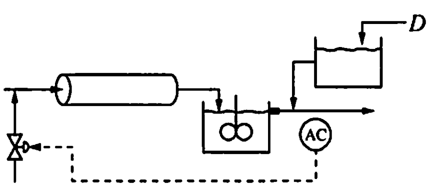


	Feedback process			Disturbance process	
Case	K_p	θ	τ	τ_d	K_d
A	1.0	1.0	1.0	1.0	2.0
B	1.0	4.0	4.0	1.0	2.0
C	1.0	0.5	1.5	1.0	2.0
D	0.1	0.5	1.5	1.0	2.0
E	1.0	0.5	1.5	4.0	2.0

The feedback control systems could be simulated to determine the performance for each. The selection of the best performing design would be straightforward, but the total effort would be substantial. In this example, the limiting (best possible) performances will be evaluated using equations (13.9) and (13.10) as a basis for selecting the best design. The results of the calculations are given in the following table.

Case	Minimum IAE equation (13.9) (smallest is best)	Minimum $\lvert E_{max} \rvert$ equation (13.10) (smallest is best)	Ranking (1 = best)
A	1.85	3.15	4
B	15.10	4.90	5
C	0.55	1.95	2 (tied)
D	0.55	1.95	2 (tied)
E	0.15	0.59	1

The rankings of the original four cases agree with the conclusions in Example 13.1. All of these have the same disturbance dynamics, so that the performance ranking depends entirely on the feedback dynamics. Since cases C and D have the smallest dead time and fraction dead time, they provide the best performance from among the original cases A to D. Case E has the same feedback dynamics as cases C and D, but it has slower disturbance dynamics. Slower disturbance dynamics are favorable, because feedback compensation has more time to correct for the disturbance before a large deviation from set point occurs. The performance measures indicate that case E should give substantially better performance than the other designs for this step disturbance. Simulations with realistic PID controller tuning confirm these conclusions, which are based on the theoretically best possible performance.



EXAMPLE 13.7.

As a result of Example 13.6, we have selected the case E process design. The customers of the product have stated that they will not accept the product if it ever deviates more that ± 0.40 units from the desired value, i.e., the controller set point. How does our design measure up to this demand?

The results table in Example 13.6 shows that the smallest possible maximum deviation is 0.59, which is *larger* than the maximum allowable violation. Since this is the best possible performance—with feedback control—we know that we should not investigate alternative PID tuning or alternative feedback control calculations. We know that we must change the structure of the problem. Possible solutions include (1) reducing the magnitude of the disturbance in an upstream process (always a good concept), (2) making the feedback process faster, (3) making the disturbance process slower, or (4) inventing a control approach different from feedback. In this example, we will investigate (3) by modifying the disturbance process. (In the next few chapters, we will develop new control approaches that might be less expensive.)

The simplest change to the disturbance process would be an increase in the volume of the mixing tank that would increase the disturbance time constant. From equation (13.10), the minimum disturbance time constant to achieve the required performance (minimum $\lvert E_{max} \rvert \leq 0.40$) is about 6.0. However, this calculation assumes the best possible feedback compensation; therefore, a larger disturbance tank volume would be expected for realistic feedback control. A few

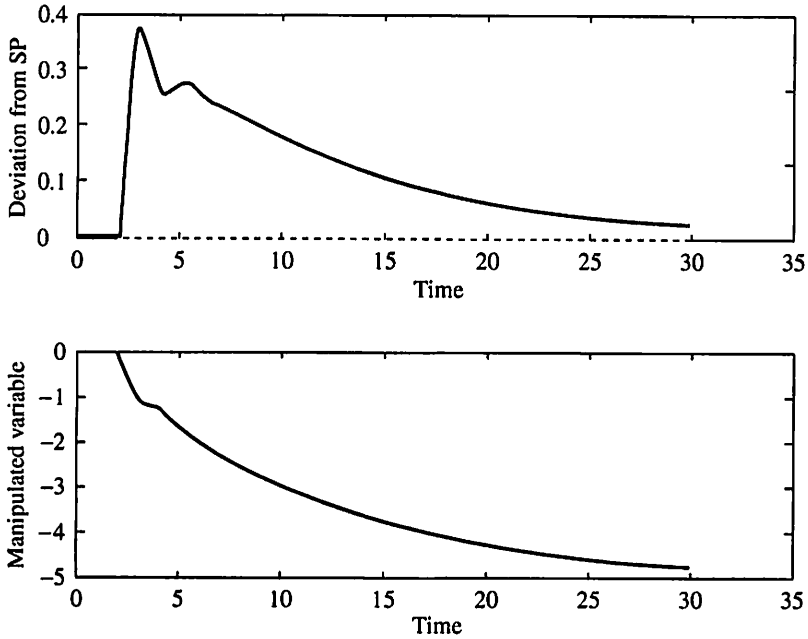


FIGURE 13.13

Disturbance response of the case E process in Example 13.7 modified to have $\tau_d = 10$.

simulations with PI control and Ciancone tuning ($K_c = 1.7$ and $T_I = 1.3$) found that a disturbance time constant of 10 was just large enough to achieve the desired control performance. The dynamic response to the disturbance for a disturbance time constant of 10 is shown in Figure 13.13. As expected, the behavior of the controlled variable with a realistic PI controller is not as good as with the optimal controller; as a result, the disturbance time constant had to be increased substantially to obtain the desired performance. The wise engineer would evaluate the likely errors in the plant models and further increase the disturbance mixing tank volume to account for these uncertainties.

The preceding discussion and examples demonstrate that both feedback and disturbance process dynamics influence control performance. *Fast* feedback dynamics and *slow* disturbance dynamics favor good performance. Understanding this difference is crucial when designing plants with favorable dynamic behavior.

The Effect of Inverse Response

Inverse response is an important characteristic of the feedback process dynamics that, when it exists, has a major effect on control performance. The reasons why inverse responses occur are explained in Section 5.4 on parallel systems, and some process systems that have parallel structures are presented and modelled in Appendix I. The process considered here is modelled in Example I.2. In that example, the parallel process structure resulted in the concentration first increasing, then decreasing in response to a step increase in the solvent flow rate. (The reader may want to review this example before proceeding.) Clearly, such a process is difficult to control, because the initial response of the controlled variable is in the "wrong" direction. The initial inverse response imposes a limit to the achievable control performance in a way similar to dead time.

EXAMPLE 13.8.

The inverse response process, the reactor in Example I.2, is shown in Figure 13.14 with the proposed feedback control system. Determine the control performance for this system in response to a step change in the set point of a PI controller.

The model for this process, linearized about the initial steady state, is repeated here; however, this model is not exact for the transient considered, because the gain and time constants depend on the flow of solvent, which changes through the transient:

$$G_p(s) = \frac{-1.66(-8.0s+1)}{(8.25s+1)^2} \tag{13.11}$$

The tuning for the PI controller was determined by trial and error to be $K_c = -0.45\ \text{m}^3/\text{min(mole/m}^3)$ and $T_I = 13.0$ min, which resulted in the transient response in Figure 13.15. This transient was evaluated by a numerical solution of the nonlinear differential equations. The control performance is less than ideal, because the initial response of the controlled variable is inverse to the change in the set point. However, the response is stable, returns to the set point, and is "well behaved" (i.e., not unduly oscillatory or slow to return to the set point).

> It is important to recognize that this second-order process *without dead time* cannot be controlled tightly, because of the inverse response, regardless of the feedback control algorithm.

Again, we see the influence of feedback dynamics on control performance.

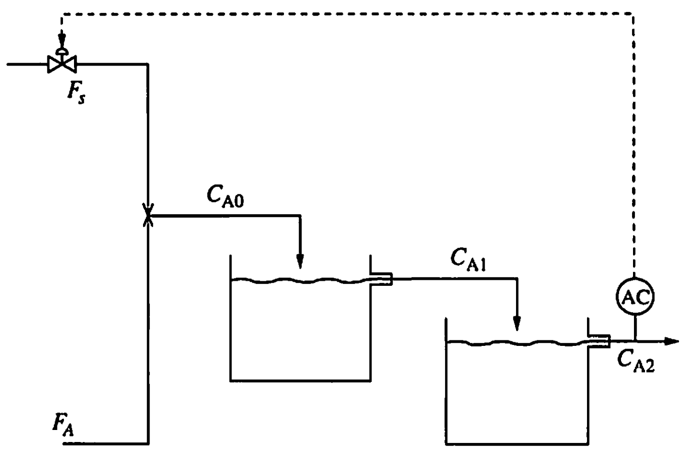


FIGURE 13.14

Feedback control design for Example 13.8.

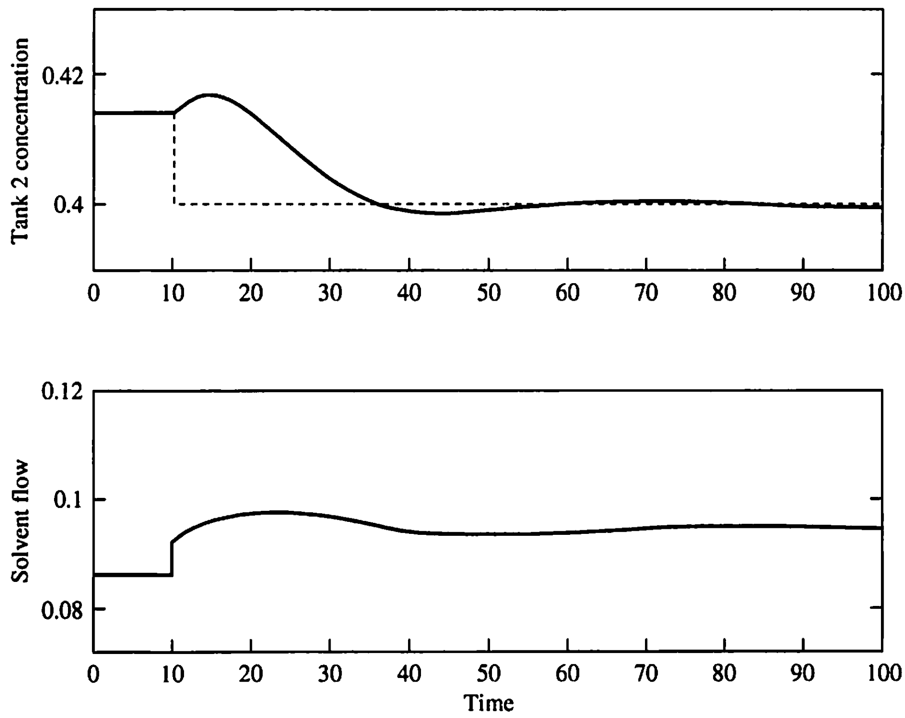


FIGURE 13.15

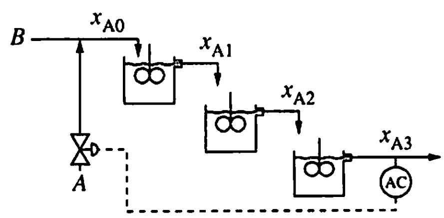
Closed-loop response of the inverse response process in Example 13.8.

Model Requirements for Predicting Control Performance

Throughout this book, we have monitored the effects of modelling errors on design decisions such as tuning and on the resulting control performance. Here the effects of modelling errors on the accuracy of control performance predictions are considered. Two linear models for the three-tank mixing process have been developed; one involves a third-order system, and the other involves a first-order-with-dead-time approximation. How well does the performance predicted using the approximate model compare with the performance using the "exact" third-order model? To answer the question for this example, the closed-loop frequency responses have been calculated for both cases. The controller is a PI algorithm with the tuning constants from Example 9.2 (with the small derivative time set to zero). The closed-loop transfer functions for the two cases are as follows:

Exact third-order model.

$$\frac{CV(s)}{D(s)} = \frac{\dfrac{1}{(5s+1)^3}}{1 + \dfrac{0.039}{(5s+1)^3} 30 \left(1 + \dfrac{1}{11s}\right)} \tag{13.12}$$



Approximate first-order-with-dead-time model.

$$\frac{CV(s)}{D(s)} = \frac{\dfrac{1e^{-5.5s}}{(10.5s+1)}}{1 + 0.039 \dfrac{e^{-5.5s}}{(10.5s+1)} 30 \left(1 + \dfrac{1}{11s}\right)} \tag{13.13}$$

The results of the analysis are plotted in Figure 13.16. The approximate first-order-with-dead-time model represents the system with sufficient accuracy to predict the control performance, especially for the low-frequency disturbances, which is the range for which feedback control is designed and effective. The predictions differ in the high-frequency range, but they both predict very good disturbance attenuation. The approximate model leads to some error in the region of the resonance peak; however, both models identify the proper resonance frequency and properly predict that feedback is not effective in this frequency region.

The results of this example on control performance, along with Examples 9.2 and 9.3 on tuning and Example 10.17 on stability analysis, lead to a very important conclusion:

An approximate first-order-with-dead-time model typically provides sufficient accuracy for single-loop control tuning and performance analysis when the open-loop process has an overdamped, sigmoidally shaped response between the manipulated and controlled variables.

Since many processes have such well-behaved dynamic responses, the first-order-with-dead-time models are used frequently in the process industries.

The topics in this section demonstrate some key limitations imposed on control performance by process dynamics and provide some quantitative estimates of how various process parameters affect performance. From these results, it becomes clear that many deficiencies in control performance cannot be corrected by improving the single-loop control algorithm or tuning. Finally, the sensitivity of control design methods to modelling errors has been analyzed, and the results in this section, in conjunction with previous chapters, confirm the usefulness of approximate models.

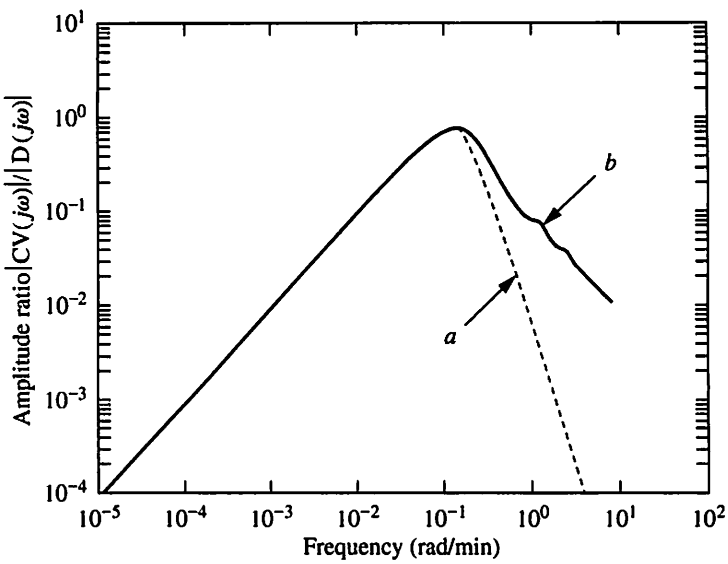


FIGURE 13.16

Comparison of closed-loop frequency response for (*a*) exact third-order model, equation (13.12), and (*b*) approximate process model, equation (13.13).

13.6 CONTROL SYSTEM FACTORS INFLUENCING CONTROL PERFORMANCE

The goal of the control instrumentation and algorithm is to achieve, as closely as is practically possible, the best control performance (for the controlled and manipulated variables) for the existing process dynamics. The effect of controller algorithm and tuning constants on the system's stability has been covered extensively in Chapters 9 and 10 and will not be repeated here. Suffice it to say that the controller tuning is selected to provide a compromise that gives acceptable behavior over a range of process dynamics. Several other important control system factors are discussed in this section.

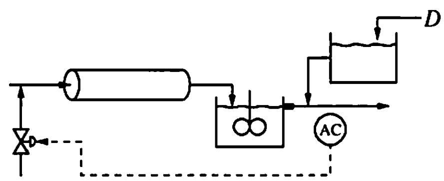
Manipulated-Variable Behavior

As emphasized in Chapter 9, the behavior of the manipulated variable is also considered when evaluating control system performance. The effect of feedback control can be determined from the block diagram in Figure 13.1.

$$\frac{\mathrm{MV}(s)}{D(s)} = \frac{-G_d(s)G_s(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)} \tag{13.14}$$

The numerator includes the product of the disturbance and controller transfer functions. As the controller tuning is selected for more aggressive control (i.e., the gain is increased or integral time decreased), the magnitude of the manipulated-variable variation is increased. In contrast, maintaining the controlled variable close to its set point requires aggressive control, as limited by feedback dynamics. Thus, the tuning is often selected as a compromise of these two concerns, manipulated- and controlled-variable performance.

EXAMPLE 13.9.



Evaluate the frequency response of the controlled and manipulated variables for the system in Example 13.1, case C. Evaluate three values of the controller gain relative to the base case: (*a*) 75%, (*b*) 100%, and (*c*) 125%.

The magnitude of the controlled variable is determined from equation (13.2), and the magnitude of the manipulated variable is determined from the following equation:

$$\frac{|\mathrm{MV}(j\omega)|}{|D(j\omega)|} = \left|\frac{G_d(j\omega)G_s(j\omega)G_c(j\omega)}{1 + G_p(j\omega)G_v(j\omega)G_c(j\omega)G_s(j\omega)}\right| \tag{13.15}$$

The results are given in Figure 13.17*a* and *b*. Note that the manipulated-variable variation at low frequencies is nearly independent of the controller gain, since the manipulated variable is adjusted slowly, in quasi-steady state, in response to the disturbance magnitude. However, at higher frequencies a smaller controller gain results in a smaller manipulated-variable magnitude (variation). As expected, the smaller controller gain also results in an increased controlled-variable magnitude (variation).

Sensor and Final Element Dynamics

The dynamics of the final control element, usually but not always a valve, and the sensor appear in the feedback path. Therefore, they influence the stability and

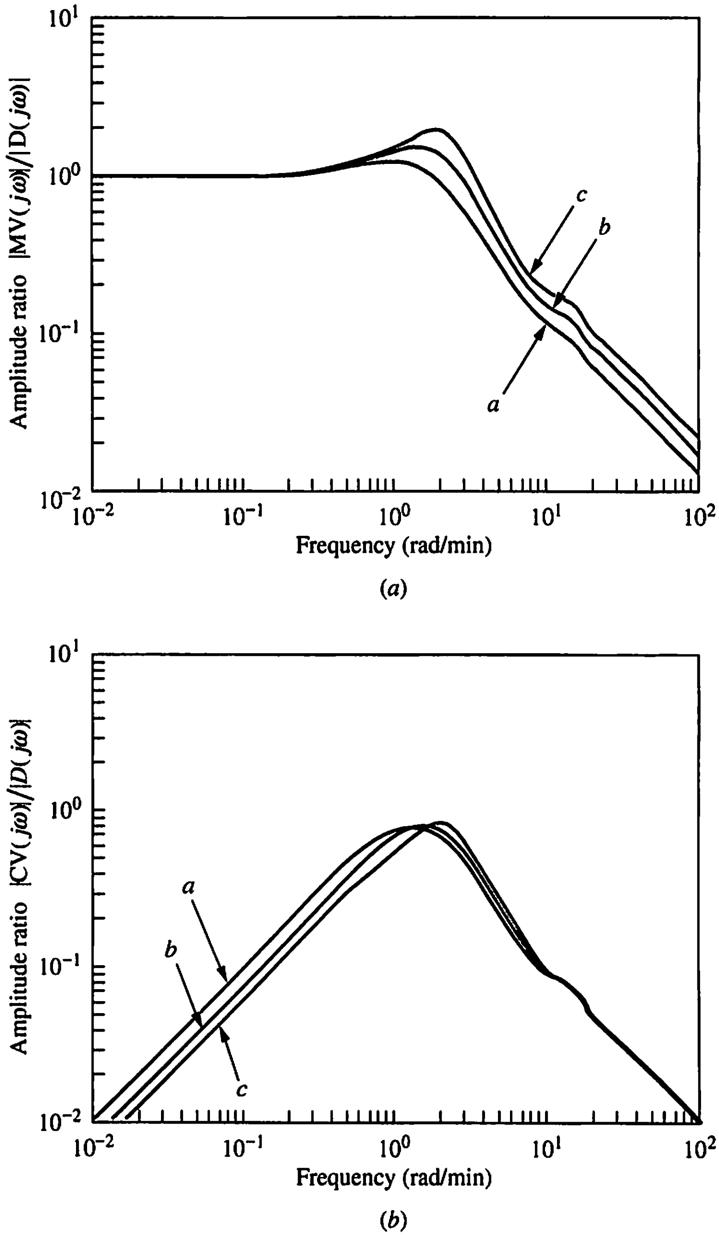


FIGURE 13.17

Amplitude ratios for disturbance input for Example 13.9: (*a*) of manipulated variable; (*b*) of controlled variable.

control performance. The closed-loop transfer function, including the instrument elements, for the system was derived in Chapter 7 and is repeated here:

$$\frac{CV(s)}{D(s)} = \frac{G_d(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)} \tag{13.16}$$

EXAMPLE 13.10.

Calculate the frequency response of the controlled variable to a disturbance input for the system in Example 7.1, case A, (*a*) when the sensor and final element dynamics are as given in the Example, and (*b*) when these dynamics are negligible (i.e., all instrument dead times and time constants are reduced to zero, so that the only significant dynamics in the feedback path are from the process). For both cases, the disturbance time constant is 3 minutes. The models for the two situations are given below.

Example 13.10(*a*)

$$G_p(s) = \frac{1.84e^{-s}}{(0.5s+1)(1.5s+1)(3s+1)(10s+1)(0.51s+1)(s+1)} \qquad G_d(s) = \frac{1.0}{(3s+1)}$$

Example 13.10(*b*)

$$G_p(s) = \frac{1.84e^{-s}}{(3s+1)} \qquad G_d(s) = \frac{1.0}{(3s+1)}$$

The controller tuning has to be determined individually for (*a*) and (*b*). The dynamics can be approximated from the process reaction curves in Figure 7.3*a* using the process reaction curve graphical Method II, and the tuning can be calculated from the Ciancone correlations.

	K_p	θ	τ	K_c	T_I
Example 13.10(*a*)	1.84	5.5	13.5	0.65	13.3
Example 13.10(*b*)	1.84	1.0	3	0.65	2.8

The results of the frequency response calculations are given in Figure 13.18. Clearly, the control performance is better for (*b*), where the instrumentation dynamics are negligible, because the instrumentation dynamics in (*a*) are substantial compared with the process.

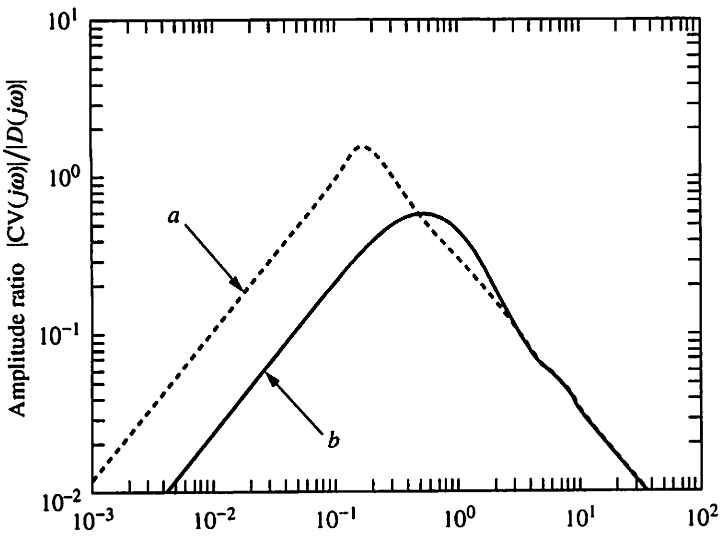


FIGURE 13.18

Amplitude ratio of controlled variable to disturbance for Example 13.10.

Recall that the dynamic model determined through empirical identification includes all elements in the feedback path, $G_p(s)G_v(s)G_c(s)G_s(s)$. When the control system uses the same instrumentation, the identified model provides the information needed for tuning and control performance assessment.

Digital PID Controllers

The PID algorithm can be implemented in a digital, or discrete manner, where the calculation is performed periodically. The effects of the execution period on tuning and control performance were covered in Chapter 11, where $\Delta t/(\theta + \tau)$ was identified as the parameter indicating the change from a continuous system. When this parameter is small, approximately 0.05, the system behavior is similar to that with a continuous controller; as the parameter increases, the control performance degrades from that achieved with a continuous controller. The digital control system can be easily simulated by executing the appropriate number of process simulation time steps between successive controller executions to provide an accurate representation of the process dynamics. The magnitude of the controlled variable in response to a sine input (i.e., the amplitude ratio of the frequency response) can be obtained; the calculations require mathematical methods for discrete systems (z-transforms) covered in this book in Appendix L and in Ogata, 1987.

PID Mode Selection

With detailed analysis of controller tuning and control system performance, it is possible to discuss the selection of controller modes—proportional, integral, and derivative—for various applications. Naturally, the appropriate selection depends on the control objectives. For the vast majority of applications, zero offset is desired for steplike inputs, and an integral mode is required, as was demonstrated in Chapter 8. A few control strategies do not require zero offset, and proportional-only control is possible for these. The most common instances are some, but not all, level controllers, which are described in Chapter 18. Also, the proportional mode is nearly always used with the integral mode, because control systems with integral-only controllers tend to have slow, oscillatory dynamic responses.

Therefore, the proportional and integral modes are used for nearly all controllers, and the only choice regards the use of the derivative mode. The tuning correlations in Chapter 9 show that the derivative time (i.e., the contribution from the derivative mode) should be small for small fraction dead times and increase as the fraction dead time increases. A rationale for this trend is that the derivative is a "predictive" mode and that prediction is needed because of the dead time in the closed-loop system. A quantitative explanation is that the phase lead provided by the derivative mode allows a higher controller gain and shorter integral time, resulting in better control performance.

As previously discussed, the derivative mode amplifies high-frequency noise in the measured variable. If the difference between the noise and process response frequencies is large, the noise can be attenuated by filtering (see Chapter 12). If this is not the case, the controller derivative time must be reduced, perhaps to zero, to observe the limitation on the high-frequency variation of the manipulated variable.

EXAMPLE 13.11.

Select appropriate modes for the PID controller applied to the process shown in Figure 13.19.

LI-1 and LI-5. The feed tanks have periodic, rather than continuous, supply flows. As a result, their levels must vary with time, and their total volumes must be large enough to contain the change in inventory accumulated between supply or delivery flows. Therefore, their levels are not controlled. Level indication allows plant operating personnel to monitor the levels.

FC-1 and FC-2. Flow controllers should maintain the flows at their set points. The flow process has little dead time and a relatively noisy measurement signal. Therefore, a PI controller is used. Since the flow process is so fast, the PI is sometimes tuned with a small gain and small integral time so that it performs closer to an integral-only controller. This tuning further reduces the effects of noise.

LC-2. The reactor level influences the residence time and, therefore, the reaction conversion. The level should be maintained at its set point, but extremely rapid changes to the manipulated flow are not desirable. A PI controller is used.

TC-1. The reactor temperature is also a key variable in determining the reaction conversion. The controller would be PID or PI, depending on the fraction dead time.

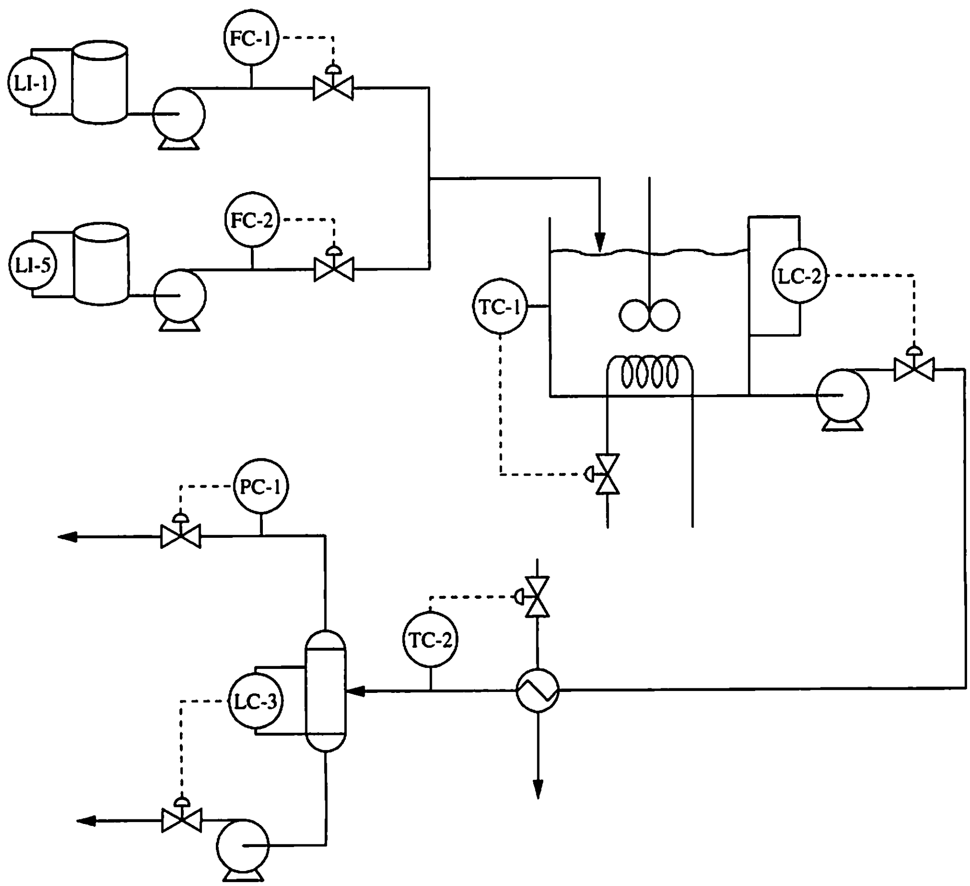


FIGURE 13.19

Schematic of process and controllers considered in Example 13.11.

TC-2. The flash drum temperature is an important variable in controlling the separation. The controller would be PID or PI, depending on the fraction dead time.

LC-3. There is no incentive to maintain the flash drum level at a specific value as long as the level remains within its allowed range. Also, flow variation to downstream units should be small. Therefore, a P-only controller could be used. A PI controller is also allowable in this case.

PC-1. The pressure of the flash drum is important for safety. It is also important for product quality, because the pressure affects the components in the flash vapor and liquid phases. The pressure dynamics should have essentially no dead time. Therefore, a PI controller is selected.

Selecting the Manipulated Variable

In Chapter 7, five criteria were presented for selecting a manipulated variable from among several candidates. Here, we apply these criteria using quantitative dynamic models that improve our ability to evaluate candidate designs and to select the best manipulated variable.

EXAMPLE 13.12.

Using the following quantitative data, select the manipulated valve for feedback control for the reactor in Figure 13.20 that will provide better control performance.

Control objective. Maintain the reactant concentration in the reactor at 0.465 mole/m^3.

Design problem. Should the feedback controller manipulate v_A or v_C to achieve good dynamic performance?

Disturbance. The reactant concentration in the solvent, $(C_A)_{SOL}$, is normally zero but can increase to 0.463 mole/m^3 in a step.

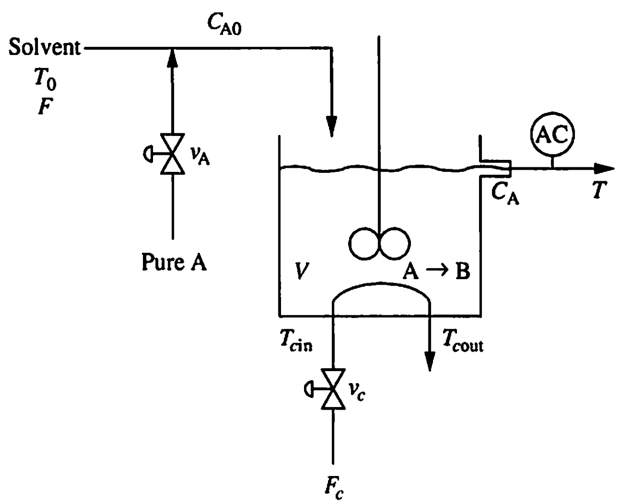


FIGURE 13.20

Chemical reactor analyzed in Example 13.12.

Model information.

1. The reaction is first-order with Arrhenius temperature dependence; $-r_A = k_0 e^{-E/RT} C_A$.
2. The reactor is well mixed, and the volume is constant.
3. Flows depend on the valve openings linearly; $F_c = K_{vc} v_C$ and $F_A = K_A v_A$.
4. Heat transfer can be modelled similarly to Example 3.7, and heat losses are negligible.
5. The heat of reaction is zero.

Data. $F = 0.085$ m³/min, $V = 2.1$ m³, $\rho = 10^6$ g/m³, $C_p = 1$ cal/(g°C), $T_0 = 150$°C, $T_{cin} = 25$°C, $F_{cs} = 0.50$ m³/min, $C_{pc} = 1$ cal/(g°C), $\rho_c = 10^6$ g/m³, $k_0 = 5.62 \times 10^7$ min^{-1}, $E/R = (15{,}000/R)K$

Steady-state operation. $(C_A)_{SOL} = 0$, $C_{A0} = 0.965$ mol/m³, $C_A = 0.465$ mol/m³, $T_s = 85.4$°C, $v_A = 50\%$, $v_C = 50\%$

A thorough analysis of the potential control designs requires information about the feedback dynamics. To provide this information, a dynamic model of the system is formulated, based on the following energy and component material balances.

$$V\rho C_p \frac{dT}{dt} = F\rho C_p (T_0 - T) - \frac{aF_c^{b+1}}{F_c + \dfrac{aF_c^b}{2\rho_c C_{pc}}}(T - T_{cin})$$

$$V\frac{dC_A}{dt} = F(C_{A0} - C_A) - Vk_0 e^{-E/RT} C_A$$

with $F_c = K_{vc} v_C$, $F_A = K_A v_A$, and $C_{A0} = (\mu_A)(F_A)/F$, with μ_A molar density in moles/m³

The equations can be linearized and the following transfer functions can be derived for the two potential feedback dynamic systems.

$$\frac{C_A(s)}{v_C(s)} = \frac{K_{FC}}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad \text{with } K_{FC} = 0.00468 \frac{\text{mole/m}^3}{\%\text{ open}}$$

$$\tau_1 = 12.4\,\text{min} \qquad \text{and} \qquad \tau_2 = 11.7\,\text{min}$$

$$\frac{C_A(s)}{v_A(s)} = \frac{K_{FA}}{(\tau_1 + 1)} \qquad \text{with } K_{FA} = 0.0097 \frac{\text{mole/m}^3}{\%\text{ open}} \qquad \tau = 12.4\,\text{min}$$

Now, the five basic criteria are evaluated for the two potential manipulated variables.

	Feedback with $v_A \to C_A$	**Feedback with $v_C \to C_A$**
1. Causal relationship	Yes, $K_{FA} \neq 0$	Yes, $K_{FC} \neq 0$
2. Automated valve available	Yes	Yes
3. Fast feedback dynamics	Stable, *first-order* system with $\tau = 12.4$ min; **this is faster!**	Stable, *second-order* system with $\tau_1 = 12.4$ and $\tau_2 = 11.7$ min; **this is slower!**
4. Able to compensate for largest disturbance	Yes, when $(C_A)_{SOL} = 0.463$, $v_A = 25\%$	Yes, when $(C_A)_{SOL} = 0.463$, $v_C = 25\%$
5. Adjust the valve without upsetting the plant	Yes, a tank of reactant is available	Yes, cooling water is available

Based on the analysis, either valve could be used for feedback control because a causal relationship exists, an automated valve is available, and the valve has sufficient range to compensate for the largest expected disturbance. The control performance would be best for the system with the *fastest feedback* dynamics, therefore, feedback using the *reactant valve*, v_A, is chosen as the better manipulated variable.

This analysis is confirmed by the dynamic responses of both feedback control systems in Figure 13.21. The PI controller tunings are

Manipulating v_A	$K_c = 200\%/(\text{mole/m}^3)$	$T_I = 3.0$ min
Manipulating v_C	$K_c = 200\%/(\text{mole/m}^3)$	$T_I = 13.0$ min

The transient responses show the concentration deviating much less from its set point when v_A is manipulated. This confirms our qualitative analysis. Naturally, the selection could be influenced by other factors like the cost of energy and potential side reactions, which are not considered in this example.

Process Design for Control Performance

Various process designs can have identical steady-state conditions but very different dynamic behavior. One aspect of dynamic behavior that significantly influences control performance is process self-regulation. Processes with strong self-regulation tend to be affected less by some disturbances and can be quickly returned to desired values. The following example shows that fundamental models provide insight that enables us to design processes with good control performance.

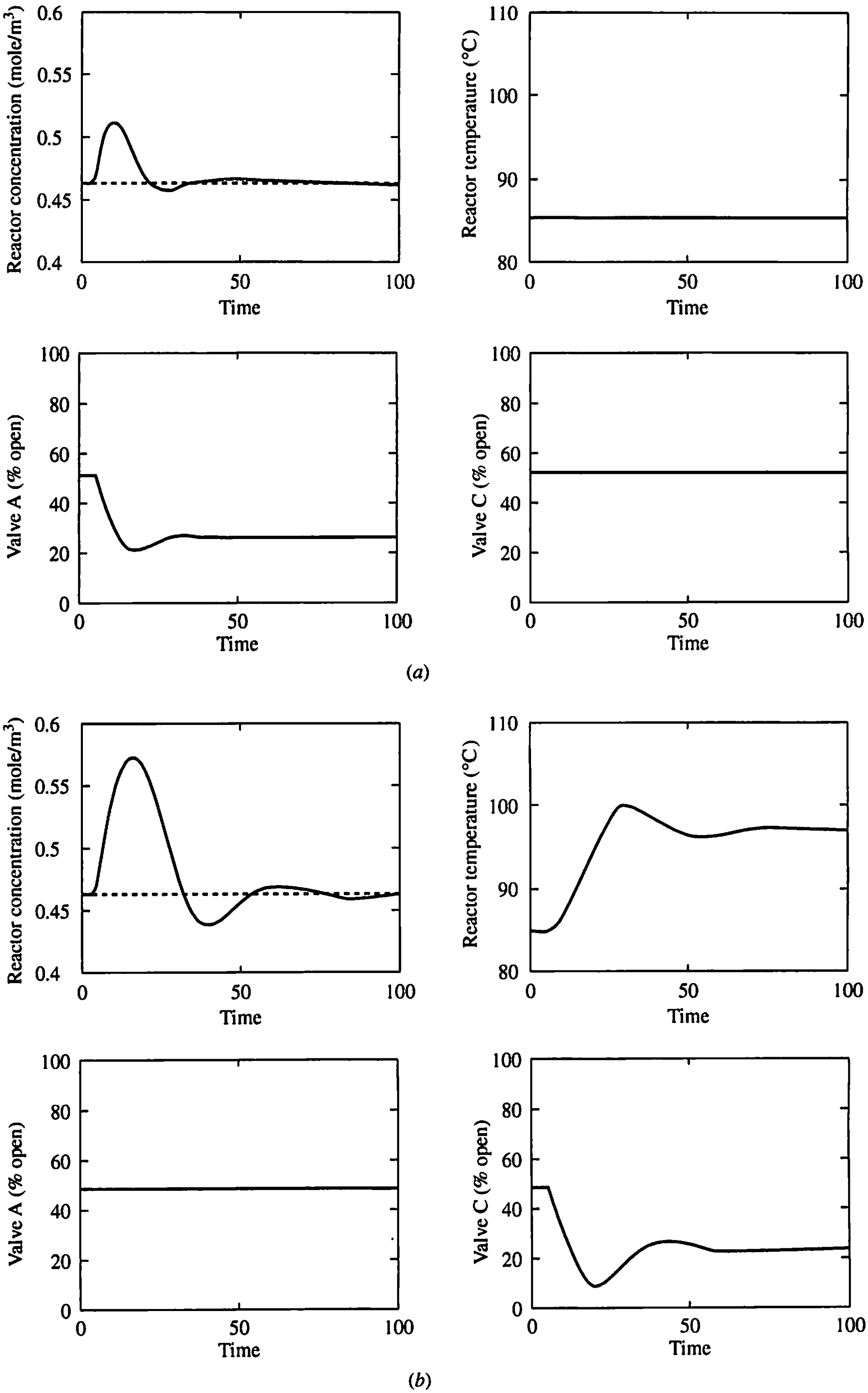
EXAMPLE 13.13.

A stirred-tank heat exchanger was modelled in Example 3.7, and the control performance of the linearized approximation was evaluated analytically in Example 8.5. The results indicated reasonably good performance, because the feedback dynamics $G_p(s)$ were first-order. However, the question remains whether the performance could be improved by simple process modifications. A reasonable goal would be to change the process so that the feedback dynamic response is faster and the controlled variable is less sensitive to disturbances. This can be achieved by increasing the "self-regulatory" nature of the process without control. For the heat exchanger, the process will be more self-regulatory if the temperature driving force for exchange is small; then, a small increase in the exchanger fluid temperature due to a feed inlet temperature increase will substantially increase the cooling duty. Naturally, the heat exchanger area must be increased to achieve the same heat transfer rate as in the base case with a smaller temperature difference.

This concept is applied to the example heat exchanger by increasing the cooling temperature from the original value of 25°C to 65°C, with a commensurate increase in the heat exchanger area. The data for this example, which is the same as the original process in Examples 3.7 and 8.5, are summarized below, and the modified data are summarized in Table 13.2.

$F = 0.085$ m^3/min	$V = 2.1$ m^3	$T_s = 85.4$°C	$\rho = 10^6$ g/m^3
$C_p = 1$ cal/(g°C)	$T_0 = 150$°C		
$F_{cs} = 0.50$ m^3/min	$C_{pc} = 1$ cal/(g°C)	$\rho_c = 10^6$ g/m^3	

The following fundamental nonlinear and linearized models can be derived for a disturbance in the inlet temperature



← Better performance by manipulating valve v_A

FIGURE 13.21

(*a*) Dynamic response of the control design in Example 13.12 manipulating v_A.
(*b*) Dynamic response of the control design in Example 13.12 manipulating v_C.

$$V\rho C_p \frac{dT}{dt} = F\rho C_p(T_0 - T) - \frac{aF_c^{b+1}}{F_c + \dfrac{aF_c^b}{2\rho_c C_{pc}}}(T - T_{cin})$$

$$\frac{T(s)}{T_0(s)} = \frac{G_d(s)}{1 + G_c(s)G_p(s)} = \frac{\left(\dfrac{K_d}{\tau_d s + 1}\right)}{1 + \left(\dfrac{K_p K_c}{\tau s + 1}\right)\left(1 + \dfrac{1}{T_I s}\right)}$$

$$\tau = \tau_d = \left(\frac{F}{V} + \frac{UA^*}{V\rho C_p}\right)_s^{-1}$$

$$K_p = \frac{\tau}{V\rho C_p}\left[\frac{-abF_c^b\left(F_c + \dfrac{a}{b}\dfrac{F_c^b}{2\rho_c C_{pc}}\right)(T - T_{\text{cin}})}{\left(F_c + \dfrac{aF_c^b}{2\rho_c C_{pc}}\right)^2}\right]_s$$

$$K_d = \left(\frac{F\rho C_p}{F\rho C_p + UA^*}\right)_s \qquad UA^* = \left[aF_c^{b+1}/(F_c + aF_c^b/2\rho_c C_{pc})\right]_s$$

The data in Table 13.2 demonstrate that the approximate linear dynamic model has two significant improvements for the modified process. First, the feedback time constant is smaller, allowing better feedback performance. Second, the disturbance gain is smaller, meaning that the same feed inlet temperature disturbance has a smaller effect on the process without control because of the stronger self-regulation. The faster feedback dynamics and smaller disturbance gain indicate that the feedback control performance should be better for the modified process. This analysis is confirmed by the results in Figure 13.22, which shows the temperature responses for the original and modified processes, and by the results summarized in Table 13.2 on control performance. The tuning was

TABLE 13.2

Data and selected results for Example 13.13

Parameter	Original value	Modified value	Comment
	Process data		
a (cal/min °C)	1.41×10^5	5.21×10^5	$UA = aF_c^b$
T_{cin} (°C)	25	65	
K_p (°C/(m³/min))	−33.9	−19.6	
τ (min)	11.9	5.93	
K_d (°C/°C)	0.52	0.24	
τ_d (min)	11.9	5.93	
	Controller data		
K_c ((m³/min)/°C)	−0.059	−0.10	$K_c K_p$ the same
T_I (min)	0.95	0.47	T_I/τ the same
	Control system performance		
IAE (°C min)	5.31	1.27	Due to smaller K_d and τ
Maximum deviation from set point (°C)	0.66	0.33	Due to smaller K_d

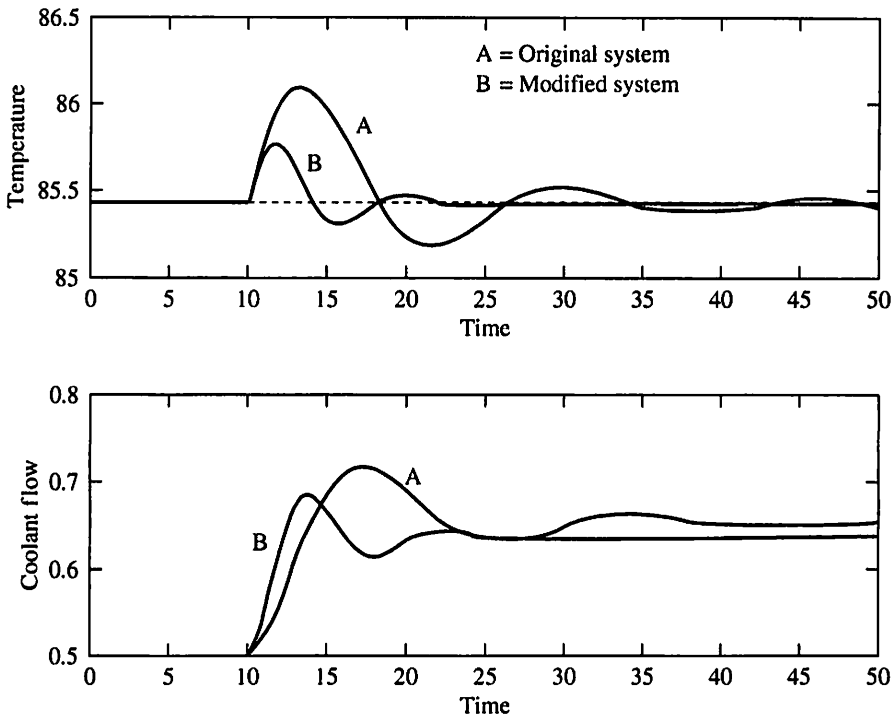


FIGURE 13.22

Transient responses for Example 13.13.

similar for both systems, adjusted to have the same values for the key dimensionless parameters K_cK_p and T_I/τ so that the manipulated-variable behavior is reasonable (and similar) for both transients. These responses were determined by numerically integrating the nonlinear differential equations for the process and controller.

The substantially improved performance for the inlet temperature disturbance has been accomplished with minor modification to the process. However, it is not without some negative impact. First, the heat exchanger area and cost have been increased. Second, the sensitivity of the process performance to disturbances in the coolant inlet temperature has increased. Thus, the best overall design and dynamic behavior must be tailored to each specific situation. This example demonstrates that strong self-regulation for key disturbances can reduce controlled variable variation and thus, improve control performance.

13.7 □ CONCLUSIONS

Two general, quantitative methods—frequency response and dynamic simulation—have been introduced for analyzing the control performance of feedback control systems. Each has specific strengths. Frequency response clearly shows the effects of the input frequency on the closed-loop performance, as indicated by the magnitude of important variables; it is applicable to stable, linear systems. Dynamic simulation provides detailed information on the performance of variables throughout a transient for any time-varying input function and can be applied to any system, linear or nonlinear. Both of these methods require extensive computation and are implemented using computer calculations.

The two quantitative analysis methods have been used to develop insights and generalizations about control performance. Many general conclusions have been developed about the effects of process and controller parameters on control performance, and they are summarized in Table 13.3.

TABLE 13.3

Summary of factors affecting single-loop PID controller performance

Key factor	Typical parameter	Effect on control performance
Feedback process gain	K_p	The key factor is the product of the process and controller gains. For example, a small process gain can be compensated by a large controller gain. Note that the manipulated variable must have sufficient range.
Feedback process "speed"	$\theta + \tau$	Control performance is always better when this term is small.
Feedback fraction process dead time	$\frac{\theta}{\theta + \tau}$	Control performance is always better when this term is small.
Inverse response	Numerator term in transfer function, $(\tau s+1)$ with $\tau < 0$	Control performance degrades for large inverse response.
Magnitude of disturbance effect	$\lvert K_d \Delta D \rvert$	Control performance is always better when this term is small.
Disturbance dynamics	τ_d	Control performance is best when the disturbance is slow (the time constant is large).
	ω_d	Feedback control is effective for low-frequency disturbances and is least effective at the resonant frequency.
	θ_d	Disturbance dead time does not influence performance.
Sensor		Measurement should be accurate. Dynamics should be fast with little noise.
Filter	$\tau_f/(\theta + \tau)$	Attenuates higher-frequency components of measurement. Reduces the variability of the manipulated variable, but degrades controlled-variable performance as filter time constant is increased.
Final element		Dynamics should be fast without sticking or hysteresis. Range should be large enough for response to demands.
Controller execution period	$\frac{\Delta t}{\theta + \tau}$	Control performance is best when this parameter is small. Continuous PID tuning correlations can be used by modifying the dead time, $\theta' = \theta + \Delta t/2$.
Controller tuning	$K_c K_p$ $\frac{T_I}{(\theta + \tau)}$ $\frac{T_D}{(\theta + \tau)}$	These terms are determined from tuning correlations based on control objectives (see Chapters 7, 9, and 10).
Modelling errors		Errors in the process model parameters lead to poorer control performance and, potentially, instability. Tuning should consider the estimate of model errors.
Limitations on manipulated variables	$\min < MV(t) < \max$	Limitations on manipulated variables reduce the operating window (the range of achievable conditions). An active limit would cause steady-state offset from the set point.

The analysis of controller modes, tuning, and stability in Chapters 8 through 10 emphasized the feedback process dynamics. In fact, it was demonstrated in Chapter 10 that the stability of linear systems is independent of the type of input, so long as it is bounded. In contrast:

> **Control system performance depends on the dynamics of both the feedback and the disturbance processes and depends critically on the frequency and magnitude of the disturbance.**

Although generally giving good performance, the PID controller does not provide the best performance in all cases. The performance of a single-loop PID control system can be improved in some cases by using additional measurements, modified PID algorithms, or entirely new feedback algorithms. Some of the most successful enhancements for single-loop control are described in Part IV of this book.

REFERENCES

Astrom, K., "Towards Intelligent PID Control," *Automatica, 28, 1*, 1–9 (1992).

Astrom, K., *Introduction to Stochastic Control Theory*, Academic Press, New York, 1970.

Astrom, K., and B. Wittenmark, *Computer Controlled Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1984.

Edgar, T., and J. Hougen, "Other Synthesis Methods: Feedback Controller Design," in Edgar, T. (ed.), *AIChE Modular Instruction, Series A, Vol. 2*, AIChE, New York, 1981, 53–61.

Kraniauskas, P., *Transforms in Signals and Systems*, Addison-Wesley, Reading, MA. 1992.

MATLAB™, The MathWorks, Inc., Cochituate Place, 24 Prime Park Way, South Natick, MA, 01760.

Newton, G., L. Gould, and J. Kaiser, *Analytical Design of Feedback Controls*, Wiley, New York, 1957.

Ogata, K., *Digital Control Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

ADDITIONAL RESOURCES

A nice introduction to a rigorous analysis of stochastic control systems, like those in Figure 13.11, is given by

Koenig, D., *Control and Analysis of Noisy Processes*, Prentice-Hall, Englewood Cliffs, NJ, 1991.

The principles in this chapter have been combined with dimensional analysis to develop correlations used to predict control performance based on the process dynamics. Useful results are reported in the following references:

Jeffreson, C., "Controllability of Process Systems," *IEC Fund., 15, 3*, 171–179 (1976).

Lopez, A., P. Murrill, and C. Smith, "Tuning PI and PID Digital Controllers," *Instr. and Contr. Sys.*, 89–95 (February 1969).

The performance of control systems can be monitored and diagnosed with the goal of improving performance.

Smith, K., *A Methodology for Applying Time Series Analysis Techniques*, ISA paper no. 92-0087, 1992.
Weinstein, B., "A Sequential Approach to the Evaluation and Optimization of Control System Performance," *Proc. Amer. Contr. Conf.*, 2354–2358 (1992).

Many examples of control performance have been published, such as the following:

Bellingham, B., *Energy Conservation via Distributed Process Control*, ISA paper no. 85-0716, 1985.
Leonard, D., and T. Kehoe, "Automatic Control Ups Heater Combustion Efficiency," *Oil Gas J.*, 134–138 (September 1981).
Marlin, T., J. Perkins, G. Barton, and M. Brisk, *Advanced Process Control Applications—Opportunities and Benefits*, Instrument Society of America, Research Triangle Park, NC, 1987.
Smith, D., W. Stewart, and D. Griffin, "Distill with Composition Control," *Hydrocarbon Proc.*, *57, 2,* 99–107 (1978).

Control performance depends on all elements in the feedback loop and the disturbance path. The following questions require you to (1) apply general principles to evaluate designs and (2) apply quantitative analysis to answer analytical or numerical questions.

QUESTIONS

13.1. The mixing process in Figure Q13.1 is to be analyzed in this question. The concentration at the outlet is controlled by adjusting a mixing stream at the inlet of three tanks. The main disturbance is the concentration of a stream flowing through a long pipe and a single stirred tank. Assume that in the base case the feedback PI controller is well tuned. For each of the following changes (*a*) through (*f*) from the base case answer the following questions and explain your answer.

(i) How should the two tuning constants be changed (increased, decreased, or unchanged) to maintain good control performance?

(ii) After the tuning has been adjusted, when necessary, how would the control performance, as measured by maximum deviation of the controlled variable in response to a step disturbance, differ from the base case (larger, smaller, same value)? Hint: It would help to identify the feedback and disturbance paths, which elements are in each, and how each is affected by the changes considered.

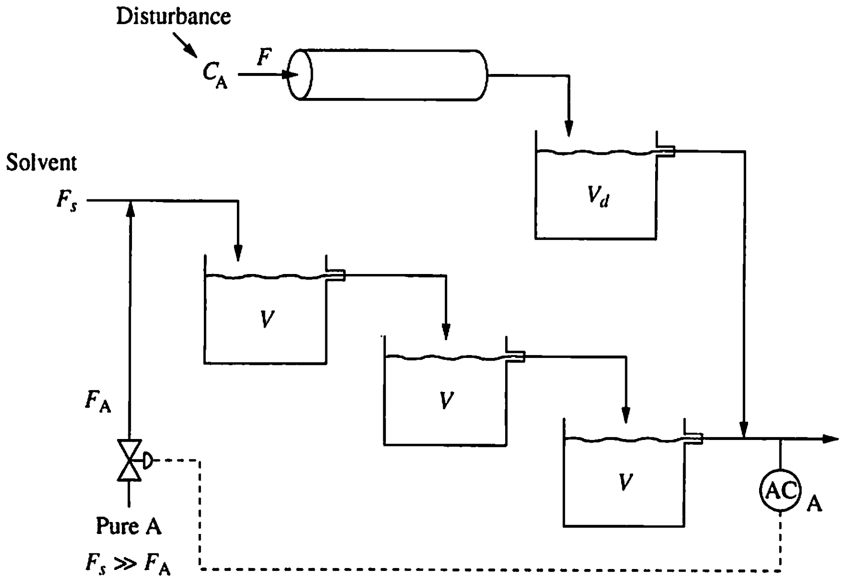


FIGURE Q13.1

Process changes (considered individually)

(*a*) The volume of each of the three tanks, V, is increased by 50%.
(*b*) The volume of the single tank, V_d, is increased by 50%.
(*c*) The initial operating condition (controller set point) is increased from 1% to 2% of A in the product.
(*d*) The length of pipe is doubled.
(*e*) The maximum flow capacity of the control valve is doubled.
(*f*) The solvent flow, F_s, is reduced by 50%.

13.2. Five mixing process designs, all having the process structure shown in Figure Q13.2, are to be analyzed in this question. The concentration at the outlet is controlled by adjusting a mixing stream at the inlet. The main disturbance is the concentration of a stream flowing through a pipe and a single stirred tank. The key parameters for each design are given in the following table, with all values in minutes.

Design	$L_1A/(F_A + F_S)$	$V_1/(F_A + F_S)$	$V_2/(F_A + F_S)$	$L_d/(F/A)$	$V_d/(F)$
I	1.0	1.0	1.0	1.0	1.0
II	0.5	1.0	1.0	1.0	1.0
III	1.0	0.5	1.0	1.0	1.0
IV	1.0	1.0	1.0	0.5	1.0
V	1.0	1.0	1.0	1.0	0.5

Rank the five designs from best to worst control performance in response to a step disturbance in C_A shown in the figure. Maximum deviation of the controlled variable from its set point is the measure of control performance. Assume that the feedback control system in the figure is used without

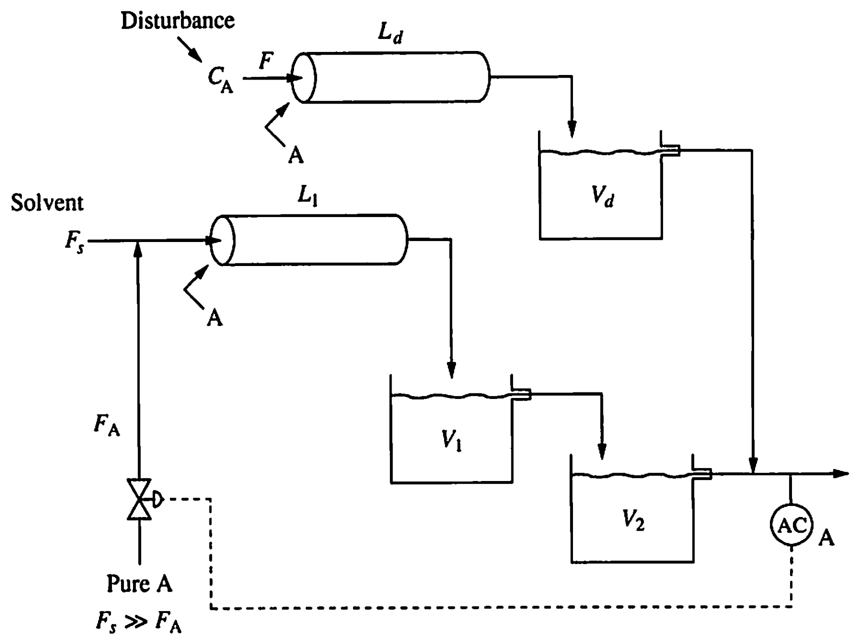


FIGURE Q13.2

change, but properly retuned for each plant. Hint: It would help to identify the feedback and disturbance paths, which elements are in each, and how each is affected by the process designs considered.

13.3. Assume that the process transfer function $G_p(s)$ used in deriving equations (13.9) and (13.10) is unchanged but that the disturbance transfer function was modified to second-order of the form that follows. Derive expressions for the minimum values for the IAE and the maximum deviation of the controlled variable equivalent to equations (13.9) and (13.10).

$$G_d(s) = \frac{CV(s)}{D(s)} = \frac{K_d}{(\tau^2 s^2 + 2\tau\xi s + 1)}$$

13.4. In this chapter the statement is made that the integral mode is particularly effective in reducing the effect of sine disturbances with low frequencies. Evaluate this statement by comparing the closed-loop frequency responses for PI and P-only controllers in the very low-frequency region. Is the P-only controller as effective? Explain your answer.

13.5. Concerning the frequency response equation (13.3):

(*a*) Verify that the equations are correct.

(*b*) Determine the modifications for a second-order disturbance model in question 13.3 being used in place of the first-order model. How would this change affect the general shape of the closed-loop frequency response?

13.6. For the following process control designs, select the proper feedback controller modes and discuss the proper execution periods for digital implementation. (*a*) Figure Q7.6, (*b*) Figure 2.2, and (*c*) Figure Q1.9 (those designs for which feedback control is possible).

13.7. (*a*) A plant with the process configuration of Figure 13.4 is analyzed in this question. Calculate closed-loop frequency responses of the controlled-variable response to a disturbance. The plant transfer functions follow with all time units in minutes, and the controller algorithm is a PI, with tuning to be determined by you. You may use equation (13.3) or use a computer program to perform the complex manipulations.

$$G_p = \frac{2.2e^{-3s}}{1+3.4s} \qquad G_d = \frac{1.0}{1+2s}$$

(*b*) The process requires a deviation from set point of less than 1.0 for the dominant disturbance, which has a magnitude of 1.5 at a frequency of 0.40 rad/min. Determine whether the PI controller can achieve the performance. If not, how should the disturbance and process feedback transfer functions be changed to satisfy the control objective?

(*c*) How would the answers to parts (*a*) and (*b*) of this question change if the disturbance transfer function $G_d(s)$ had an additional dead time of 3 min?

13.8. (*a*) In your own words, describe why processes with large dead times are difficult to control.

(*b*) Sketch a typical closed-loop frequency response and explain the three major sections of the curve at low, intermediate, and high frequencies. Perform this exercise for both set point and disturbance inputs.

(*c*) As discussed in previous chapters and reiterated here, controller tuning is selected to be somewhat conservative to ensure stability as the process dynamics change. Discuss how this tuning practice influences controller performance.

(*d*) Place each of the following factors in one of two categories, labelled "favorable" and "unfavorable" for control performance: disturbance frequency near critical frequency; small fraction dead time; large disturbance dead time; large process steady-state gain; ratio of digital execution period to feedback dynamics greater than 0.20; detuning controller gain for robustness; large value of $(\theta + \tau)$.

13.9. Open-loop responses between the manipulated and controlled variables for four potential process designs are given in Figure Q13.9, all having the same scales.

(*a*) Rank the processes for the expected control performance for set point changes.

(*b*) Rank the processes for the expected control performance for disturbance response.

13.10. Based on the model of the feedback process, how would the control performance change for the system in Example 8.5 for each of the following changes, made individually, to the initial steady-state operating conditions? Calculate the modification of tuning in response to the operating condition change and assume that this tuning change has been made.

(*a*) Determine the PI tuning that would give "good" control performance for the initial plant operating conditions in Example 8.5.

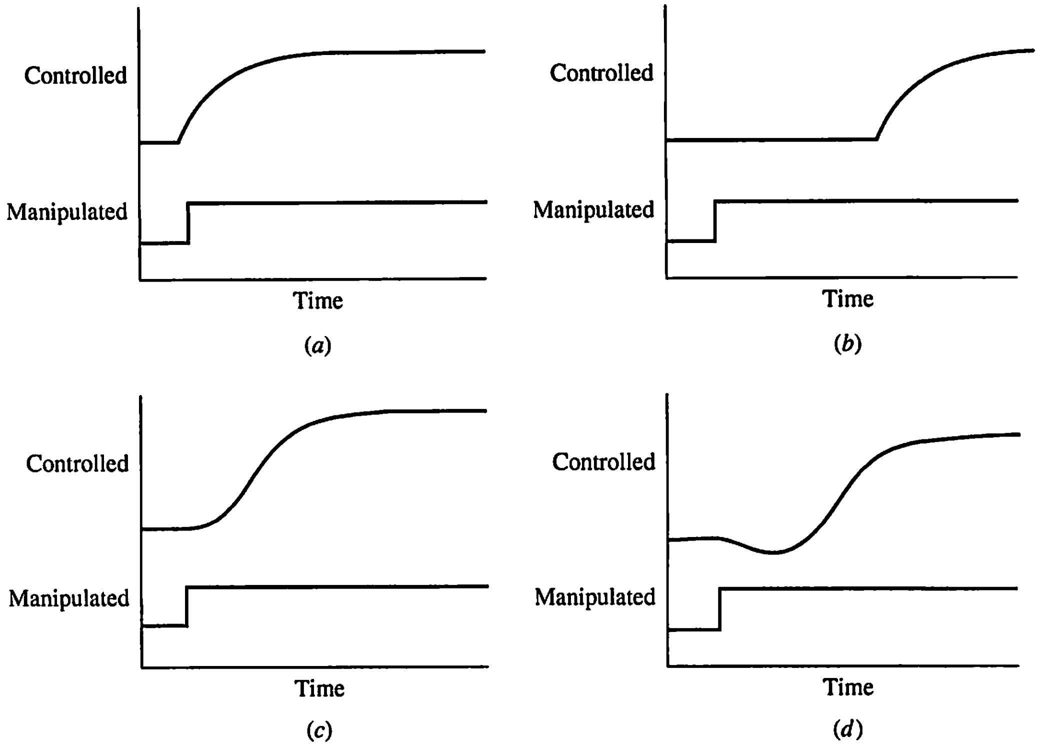


FIGURE Q13.9

(*b*) The flow through the heat exchanger is reduced from 0.085 to 0.0425 m^3/min.
(*c*) The volume in the tank is increased from 2.1 to 3.0 m^3.
(*d*) The temperature set point is changed from 85.4 to 90°C.

13.11. The closed-loop frequency response calculated using equation (13.2) for a process with the structure in Figure 13.4 and with the following process parameters is given in Figure Q13.11. Results are shown for several values of the controller gain, all with the integral time at a value of 6.0. Critically discuss these calculations and select from the three alternatives the value of the controller gain that would give the best control performance.

$$G_p(s) = \frac{-2.0e^{-2.5s}}{1+4s} \qquad G_d(s) = \frac{1.0}{1+1.8s} \qquad G_c(s) = K_c\left(1+\frac{1.0}{6s}\right)$$

13.12. (*a*) One rule of thumb for quickly estimating the standard deviation of a sample of process data is that it is equal to $\frac{1}{8}$ of the difference between the maximum and minimum values in the sample. Discuss the basis and validity of this rule of thumb.
(*b*) Apply this rule of thumb to the data in Figure 13.10*a* and *b*.
(*c*) Assume that the goal is to increase the average concentration without exceeding the value of 6.2. Evaluate the performance of the system in Figure 13.10*b* and suggest any changes to the set point that are appropriate.
(*d*) Discuss some of the factors you would consider in selecting "representative" open-loop dynamic data that could be used in estimating feed-

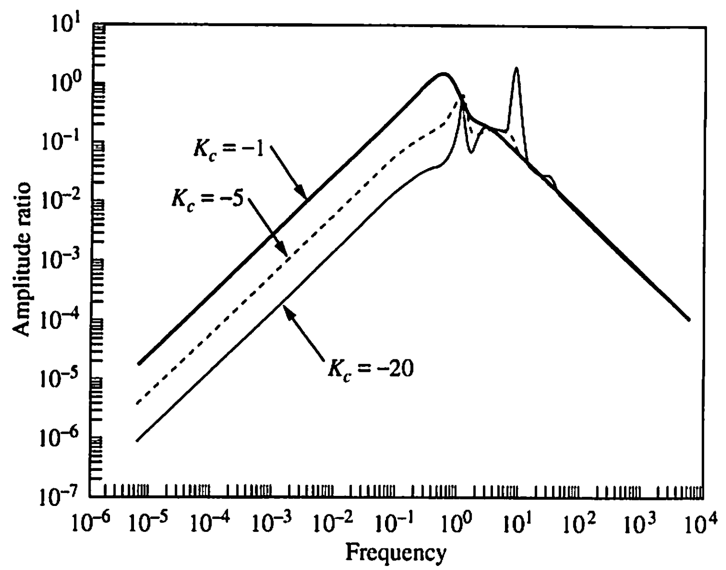


FIGURE Q13.11

back control performance for a potential PI controller by the method in Example 13.5.

13.13. Based on the model of the feedback process, answer the following questions for the three-tank system in Examples 7.2 and 9.2 for the situation in which each tank volume is increased from 35 to 105 m^3.

(*a*) Describe the control performance you would expect with the original tuning constants from Example 9.2 applied to the modified process.
(*b*) If necessary, modify the PID controller tuning.
(*c*) Compare the control performance for the original system and the modified system after tuning changes in (*b*). Consider the IAE and maximum deviation for a step inlet concentration disturbance.

13.14. Discuss how the process structure in the following systems would affect the feedback control performance: (*a*) Example I.1 (overshoot); (*b*) Example 5.5 (recycle); and (*c*) Section 3.6 (underdamped).

13.15. The tradeoff between manipulated- and controlled-variable behaviors has been discussed frequently.

(*a*) Describe the behavior of the manipulated variable for the system in Figures 9.2 and 9.3. On each figure, sketch an approximate plot of the variability of the manipulated variable, showing where the variability is high and low as a function of the variable tuning constant(s). Either of the following measures of the variability can be used.

$$\int_0^\infty \left(\frac{d\text{MV}(t)}{dt}\right)^2 dt \quad \text{or} \quad \int_0^\infty \left|\frac{d\text{MV}(t)}{dt}\right| dt$$

(*b*) Recalculate Figure 13.7 with a PID controller and discuss the difference.

13.16. The system in Example 13.10(*a*) evaluated the closed-loop amplitude ratio of the controlled to disturbance variables. For the same system, calculate the amplitude ratio of the measured variable to the disturbance. Recalling that only the measured variable is known to the plant personnel, discuss the differences in the results and their importance in analyzing plant performance.

13.17. The transfer function between the set point and the controlled variable is given in equation (13.4). Apply the following controller design method to arrive at an algorithm other than PID. Assume the input-output response is defined at some good performance [i.e., $CV(s)/SP(s) = T(s)$ is specified]. Solve for the controller transfer function that would give this performance. Discuss whether this controller can be implemented in analog or digital form.

13.18. The process design in Example 13.8 with a parallel structure is considered in this question. The concentration at the outlet of the second reactor is to be controlled as in Example 13.8, except that the flow rate of stream A (not the solvent) is to be manipulated.

(*a*) Based on the different dynamics between the manipulated and controlled variables, predict the control performance and whether it would be better than the system in Example 13.8. (Hint: The results from end-of-appendix question I.3 will help in answering this question.)

(*b*) Develop a dynamic simulation for this design, tune the feedback PI controller, and compare the control performance with Example 13.8.

13.19. The process with recycle was analyzed in end-of-chapter question 5.14. Determine the value of the recycle for which a feedback PI control system, controlling the outlet composition C_{A2} by adjusting C_{A0}, would give the best performance.

13.20. Chemical reactors were analyzed in question 5.7 for two different reaction kinetics. For both kinetics (answered separately), determine which feedback control system, controlling C_A or C_B by adjusting C_{A0}, would provide the best performance. Base your answer entirely on the feedback dynamics, not the process gain.