

# Digital Implementation of Process Control

CHAPTER

11

## 11.1 ■ INTRODUCTION

As we have seen in the previous chapters, PID feedback control can be successfully implemented using continuous (analog) calculating equipment. This conclusion should not be surprising, given the 60 years of good industrial experience with process control and given the fact that digital computers were not available for much of this time. However, digital computers have been applied to process control since the 1960s, as soon as they provided sufficient computing power and reliability. Most, but not all, new control-calculating equipment uses digital computation; however, the days of analog controllers are not over, for at least two reasons. First, control equipment has a long lifetime, so that equipment installed 10 or 20 years ago can still be in use; second, analog equipment has cost and reliability advantages in selected applications. Therefore, most plants have a mixture of analog and digital equipment, and the engineer should have an understanding of both approaches for control implementation. The basic concepts of digital control implementation are presented in this chapter.

The major motivation for using digital equipment is the greater computing power and flexibility it can provide for controlling and monitoring process plants. To perform feedback control calculations via analog computation, an electrical circuit must be fabricated that obeys the PID algebraic and differential equations. Since each circuit is constructed separately, the calculations are performed rapidly in parallel, with no interaction between what are essentially independent analog computers. Analog equipment can be designed and built for a simple, standard calculation such as a PID controller, but it would be costly to develop analog systems

for a wide range of controller equations, and each system would be inflexible: the algorithm could not be changed; only the parameters could be adjusted.

In comparison, digital computation uses an entirely different concept. By representing numbers in digital (binary) format and solving equations numerically to represent behavior of the control calculation of interest, the digital computer can easily execute a wide range of calculations on the same equipment, hardware, and basic software. Two differences between analog and digital systems are immediately apparent. First, the digital system performs its function periodically, which, as we shall see, affects the stability and performance of the closed-loop system. Often, we refer to this type of control as *discrete* control, because control adjustments occur periodically or discretely. Second, the digital computer performs calculations in series; thus, if time-consuming steps are involved in the control calculations, digital control might be too slow. Fortunately, modern digital computers and associated equipment are fast enough that they do not normally impose limitations related to execution speed.

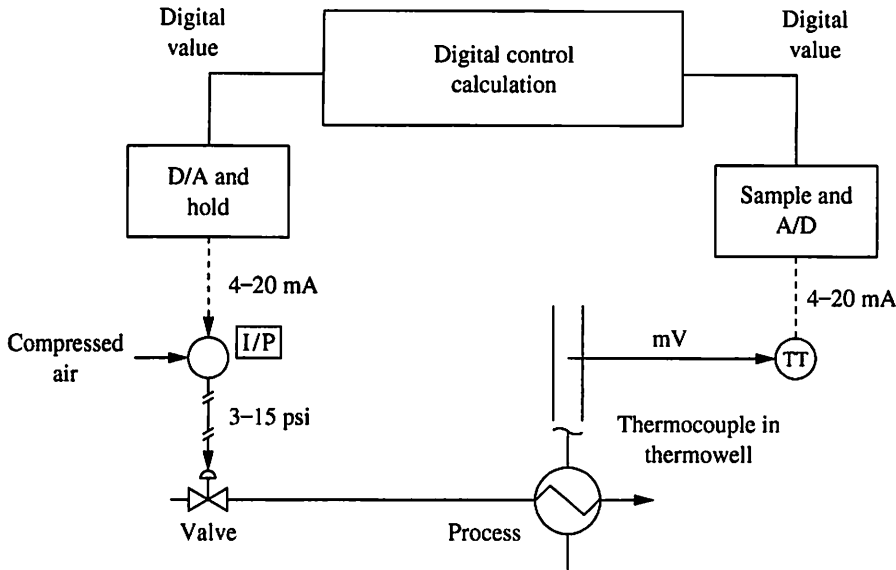
Digital computers also provide very important advantages in areas not emphasized in this book but crucial to the successful operation of process plants. One area is minute-to-minute monitoring of plant conditions, which requires plant operators to have rapid access to plant data, displayed in an easily analyzed manner. Digital systems provide excellent graphical displays, which can be tailored to the needs of each process and person. Another area is the longer-term monitoring of process performance. This often involves calculations based on process data to report key variables such as reactor yields, boiler efficiencies, and exchanger heat transfer coefficients. These calculations are easily programmed and are performed routinely by the digital computer.

The purpose of this chapter is to provide an overview of the unique aspects of digital control. The approach taken here is to present the most important differences between analog and digital control that could affect the application of the control methods and designs covered in this book. This coverage will enable the reader to implement digital PID controllers as well as enhancements, such as feedforward and decoupling, and new algorithms, such as Internal Model Control, covered later in the book.

## **11.2 ■ STRUCTURE OF THE DIGITAL CONTROL SYSTEM**

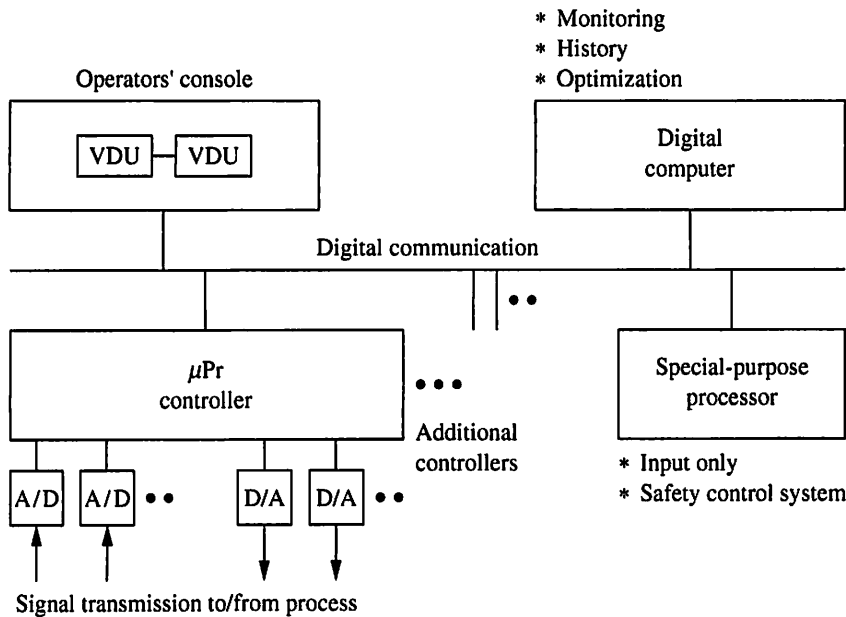
Before investigating the key unique aspects of digital control, we shall quickly review the structure of the control equipment when digital computing is used for control and display. The components of a typical control loop, without the control calculation, were presented in Figure 7.2. Note that the sensor and transmission components are analog devices and can remain unchanged with digital control calculations. The loop with digital control is shown in Figure 11.1, where the unique features are highlighted. First, the signal of the controlled variable is converted from analog (e.g., 4–20 mA) to a digital representation. Then the control calculation is performed, and finally, the digital result is converted to an analog signal for transmission to the final control element.

Process plants usually involve many variables, which are controlled and monitored from a centralized location. A digital control system to achieve these requirements is shown in Figure 11.2. Each measurement signal for control and



**FIGURE 11.1**

**Schematic of single feedback control loop using digital calculation.**



**FIGURE 11.2**

**Schematic of a distributed digital control system.**

monitoring is sent through an analog-to-digital (A/D) converter to a digital computer (or microprocessor,  $\mu Pr$ ). The results of the digital control calculations are converted for transmission in a digital-to-analog (D/A) converter. The system may have one processor per control loop; however, most industrial systems have several measurements and controller calculations per processor. Systems with 32 input measurements and 16 controller outputs per processor are not uncommon. This design is less costly, although it is somewhat less reliable, because several control loops would be affected should a processor fail.

Some data from each individual processor is shared with other processors to enable proper display and human interaction. The information exchange is performed via a digital communication network (local area network, LAN), which enables data sharing among processors and between each processor and the unit that provides operator interface, usually called the operator console. An operator console is required so that a person can monitor the process and intervene to make changes in variables such as a valve opening, controller set point, or controller status (automatic or manual). Thus, the controller set point and tuning constants must be communicated from the console, where they are entered by a person, to the processor, where the control calculation is being performed. Also, the values of the controlled and manipulated variables should be communicated from the controller to the console for display to the person. Some data that is *not* typically communicated from the individual control processors would be intermediate values, such as the integral error used in the controller calculation. The operator console has its own processor and data storage and has visual displays (video display units, VDUs), audio annunciators, and a means, such as a keyboard, for the operator to interact with the control variables. Graphical display of variables, which is easier to interpret, is used along with digital display, which is more precise. Also, variables can be superimposed on a schematic of the process to aid operators in placing data in context.

To add flexibility, more powerful processors can be connected to the local area network so that they can have access to the process data. These processors can perform tasks that are not time-critical. Examples are process-monitoring calculations and process optimization, which may adjust variables infrequently (e.g., once every few hours or shift).

Since each digital processor performs its functions serially, it must have a means for deciding which task from among many to perform first. Thus, each processor has a *real-time operating system*, which organizes tasks according to a defined priority and schedule. For example, the control processor would consider its control calculations to be of high priority, and the operators' console would consider a set point change to be of high priority. Lower-priority items, such as monitoring calculations, are performed when free time is available. An important aspect of real-time calculating is the ability to stop a lower-priority task when a high-priority task appears. This is known as a *priority interrupt* and is an integral software feature of each processor in a digital control system.

The goal, which is nearly completely achieved, is that the integrated digital system responds so fast that it is indistinguishable from an instantaneous system. Since each function is performed in series, each step in the control loop must be fast. For most modern equipment, the analog-to-digital (A/D) and digital-to-analog (D/A) conversions are very fast with respect to other dynamics in the digital equipment or the process. Each processor is designed to guarantee the execution of high-priority control tasks within a specified period, typically within 0.1 to 1 second.

When estimating the integrated system response time, it is important to consider all equipment in the loop. For example, response to a set point change, after it is entered by a person, includes the execution periods of the console processor, digital communication, control processor, and D/A converter with hold circuit and the dynamic responses of the transmission to the valve and of the valve. This total system might involve several seconds, which is not significant for most process

control loops but may be significant for very fast processes, such as machinery control.

Another important factor in the control equipment is the accuracy of many signal conversions and calculations, which should not introduce errors that significantly influence the accuracy of the control loop. The values in the digital system are communicated with sufficient resolution (16 or more bits) that errors are very small. Typically, the A/D converter has an error on the order of  $\pm 0.05\%$  of the sensor range, and the D/A converter has an error on the order of  $\pm 0.1\%$  of the final element range. In older digital control computers, calculations were performed in fixed-point arithmetic; however, current equipment uses floating-point arithmetic, so that roundoff errors are no longer a significant problem. As a result, the errors occurring in the digital system are not significant when compared to the inaccuracies associated with the sensors, valves, and process models in common use.

The system in Figure 11.2 and described in this section is a network of computers with its various functions distributed to individual processors. The type of control system is commonly called a *distributed control system* (DCS). Today's digital computers are powerful enough that one central computer could perform all of these functions. However, the distributed control structure has many advantages, some of the most important of which are presented in Table 11.1. These advantages militate for the continued use of the distributed structure for control equipment design, regardless of future increases in computer processing speed.

The major disadvantage of modern digital systems, which is not generally true for analog systems, is that few standards for design or interfacing are being observed. As a result, it is difficult to mix the equipment of two or more digital equipment suppliers in one control system.

**TABLE 11.1**

**Features of a distributed control system (DCS)**

<b>Feature</b>	<b>Effect on process control</b>
Calculations performed in parallel by numerous processors	Control calculations are performed faster than if by one processor.
Limited number of controller calculations performed by a single processor	Control system is more reliable, because a processor failure affects only few control loops.
Control calculations and interfacing to process independent of other devices connected to the LAN	Control is more reliable, because failures of other devices do not immediately affect a control processor.
Small amount of equipment required for the minimum system	Only the equipment required must be purchased, and the system is easily expanded at low cost.
Each type of processor can have different hardware and software	Hardware and software can be tailored to specific applications like control, monitoring, operator console, and general data processing.

In conclusion, the control system in Figure 11.2 is designed to provide fast and reliable performance of process control calculations and interactions with plant personnel. Clearly, the computer network is complex and requires careful design. However, the plant operations personnel interact with the control equipment as though it were one entity and do not have to know in which computer a particular task is performed. Also, considerable effort is made to reduce the computer programming required by process control engineers. For the most part, the preparation of control strategies in digital equipment involves the selection and integration of preprogrammed algorithms. This approach not only reduces engineering time; it also improves the reliability of the strategies. While distributed digital systems are the predominant structure for digital control equipment, the principles presented in the remainder of the chapter are applicable to any digital control equipment.

### 11.3 ■ EFFECTS OF SAMPLING A CONTINUOUS SIGNAL

The digital computer operates on discrete numerical values of the measured controlled variables, which are obtained by sampling from the continuous signal and converting this signal to digital form via A/D conversion. In this section, the way that the sampling is performed and the effects of sampling on process control are reviewed. As one might expect, some information is lost when a continuous signal is represented by periodic samples, as shown in Figure 11.3a through c. These figures show the results of sampling a continuous sine function in Figure 11.3a at a constant period, which is the common practice in process control and the only situation considered in this book. The sampled values for a small period (high frequency) in Figure 11.3b appear to represent the true, continuous signal closely, and the continuous signal could be reconstructed rather accurately from the sampled values. However, the sampled values for a long period in Figure 11.3c appear to lose important characteristics of the continuous measurement, so that a reconstruction from the sampled values would not accurately represent the continuous signal. The effects of sampling shown in Figure 11.3 are termed *aliasing*, which refers to the loss of high-frequency information due to sampling.

An indication of the information lost by the sampling process can be determined through *Shannon's sampling theorem*, which is stated as follows and is proved in many textbooks (e.g., Astrom and Wittenmark, 1990).

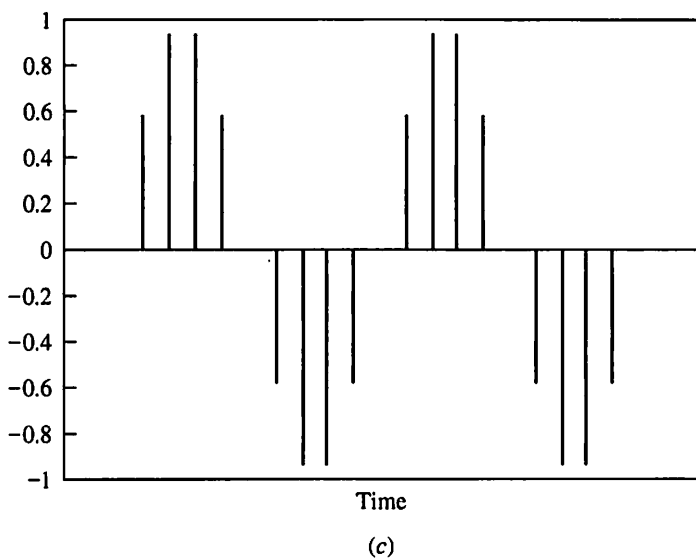
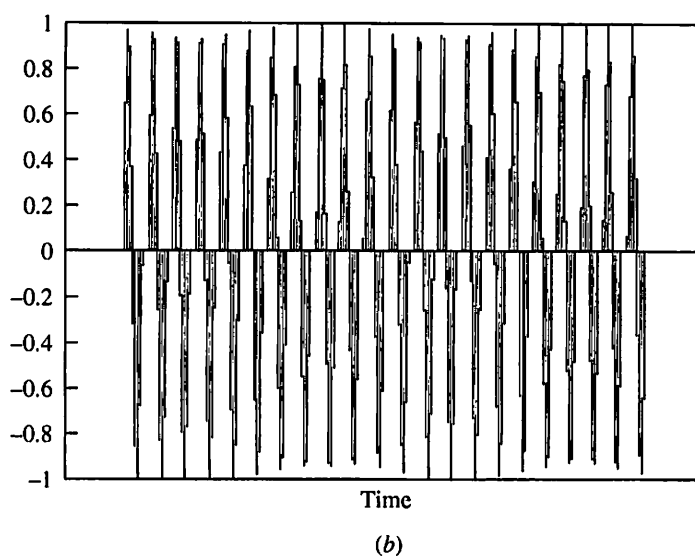
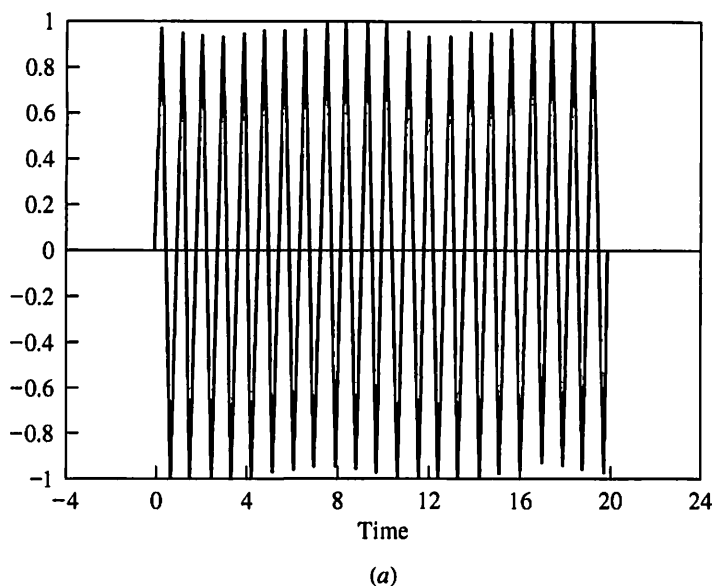
A continuous function with all frequency components at or below  $\omega'$  can be represented uniquely by values sampled at a frequency equal to or greater than  $2\omega'$ .

The importance of this statement is that it gives a quantitative relationship for how the sampling period affects the signal reconstruction. The relationship stated is not exactly applicable to process control, because the reconstruction of the signal for any time  $t'$  requires data after  $t'$ , which would introduce an undesirable delay in the reconstructed signal being available for feedback control. However, the value given by the statement provides a useful bound that enables us to estimate the frequency range of the measurement signal that is lost when sampling at a specific frequency.

**EXAMPLE 11.1.**

The composition of a distillation tower product is measured by a continuous sensor, and the variable fluctuates due to many disturbances. The dominant variations are of frequencies up to 0.1 cycle/min (0.628 rad/min). At what frequency should the signal be sampled for complete reconstruction using the sampled values?

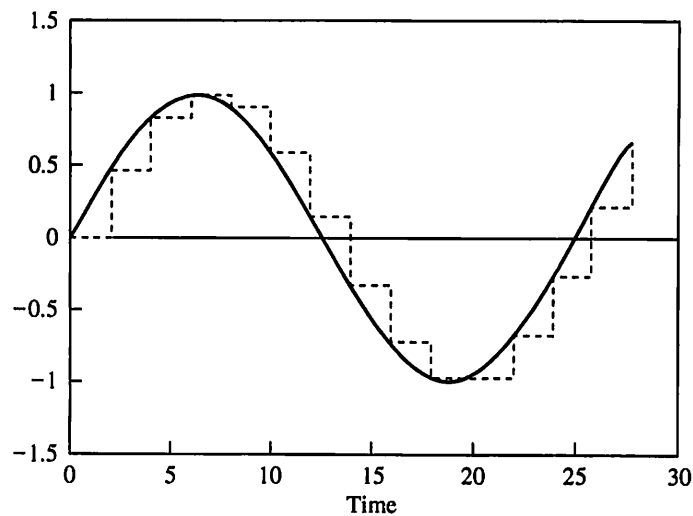
If the signal has no frequency components above 0.1 cycles/min, the sampling frequency should be 1.256 rad/min for complete reconstruction. However, most signals have a broad range of frequency components, including some at very high frequencies. Thus, a very high sampling frequency would be required for complete reconstruction of essentially all process measurement signals.

**FIGURE 11.3**

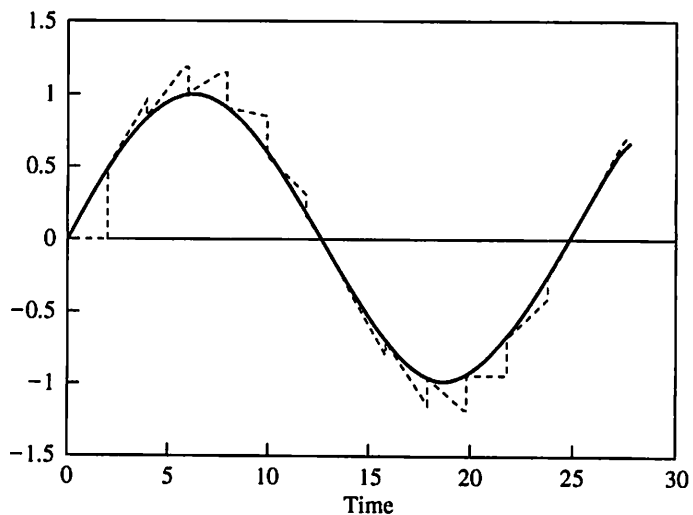
**Digital sampling:** (a) example of continuous measurement signal; (b) results of sampling of the signal with a period of 2; (c) results of alternative sampling of the signal with a period of 12.8.

Fortunately, our goal is *not* to reconstruct the signal perfectly but to provide sufficient information to the controller to achieve good dynamic performance. Thus, it is often possible to sample *much less frequently* than specified by Shannon's theorem and still achieve good control performance (Gardenhire, 1964). If the signal has substantial high-frequency components with significant amplitudes, the continuous signal may have to be filtered, as discussed in Chapter 12.

There are many options for using the sampled values to reconstruct the signal approximately. Two of the most common, zero- and first-order holds, are considered here. The simplest is the zero-order hold, which assumes that the variable is constant between samples. The first-order hold assumes that the variable changes in a linear fashion as predicted from the most recent two samples. These two methods are compared in Figures 11.4 and 11.5, where the main difference is the amplifica-

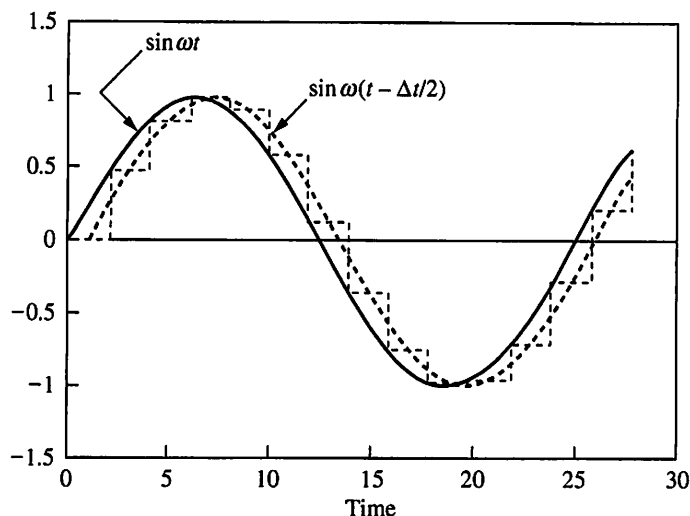


**FIGURE 11.4**  
**Zero-order hold.**



**FIGURE 11.5**  
**First-order hold.**





**FIGURE 11.6**

**Reconstruction of signal after zero-order hold.**

tion in the magnitude caused by the first-order hold. Also, the first-order hold has a larger phase lag, which is undesirable for closed-loop control. For both of these reasons, the simpler zero-order hold is used almost exclusively for process control.

The effect of the zero-order hold on the dynamics can be seen clearly if we reconstruct the original signal as shown in Figure 11.6. In the figure, the reconstructed signal is a smooth curve through the midpoint of the zero-order hold. It is apparent that the reconstructed signal after the zero-order hold is identical to the original signal after being passed through a dead time of  $\Delta t/2$ , where the sample period is  $\Delta t$  (Franklin et al., 1990). This explains the rule of thumb that the major effect on the stability and control performance of sampling can be estimated by adding  $\Delta t/2$  to the dead time of the system. Since any additional delay due to sampling is undesirable for feedback control and process monitoring, feedback control performance degrades as the process dynamics, including sampling, become slower. Therefore, the controller execution period should generally be made short.

In some cases, monitoring process operations requires high data resolution, because short-term changes in key variables can significantly influence process safety and profit. However, process monitoring also involves variables that change slowly with time, such as a heat transfer coefficient, and the data collected for this purpose does not have to be sampled rapidly.

In conclusion, sampling is the main difference between continuous and digital control. Since process measurements have components at a wide range of frequencies, some high-frequency information is lost by sampling. The effect of sampling on control performance, with a zero-order hold used for sampling, is addressed after the digital controller algorithm is introduced in the next section.

## 11.4 ■ THE DISCRETE PID CONTROL ALGORITHM

The proportional-integral-derivative control algorithm presented in Chapter 8 is continuous and cannot be used directly in digital computations. The algorithm appropriate for digital computation is a modified form of the continuous algorithm

that can be executed periodically using sampled values of the controlled variable to determine the value for the controller output. The controller output passes through a digital-to-analog converter and a zero-order hold; therefore, the signal to the final control element is changed to the result of the last calculation and held at this value until the next controller execution.

The digital calculation should approximate the continuous PID algorithm:

$$MV(t) = K_c \left[ E(t) + \frac{1}{T_I} \int_0^t E(t') dt' - T_d \frac{dCV(t)}{dt} \right] + I \quad (11.1)$$

The method for approximating each mode is presented in equations (11.2) to (11.4). In these equations the value at the current sample is designated by the subscript  $N$  and the  $i$ th previous sample by  $N - i$ . Thus the current values of the controlled variable, set point, and controller output are  $CV_N$ ,  $SP_N$ , and  $MV_N$ , respectively. The error is defined consistently with continuous systems as  $E_N = SP_N - CV_N$ .

Proportional mode:

$$(MV_N)_{\text{prop}} = K_c E_N \quad (11.2)$$

Integral mode:

$$(MV_N)_{\text{int}} = \frac{K_c(\Delta t)}{T_I} \sum_{i=1}^N E_i \quad (11.3)$$

Derivative mode:

$$(MV_N)_{\text{der}} = -K_c \frac{T_d}{\Delta t} (CV_N - CV_{N-1}) \quad (11.4)$$

The proportional term is self-explanatory. The integral term is derived by approximating the continuous integration with a simple rectangular approximation. Those familiar with numerical methods recognize that this is not as accurate an approximation as possible with other integration methods used in numerical analysis (Gerald and Wheatley, 1989). However, small numerical errors in this calculation are not too important, because the integral mode continues to make changes in the output until the error is zero. Thus, zero steady-state offset for steplike inputs is not compromised by small numerical errors. Note that all past values of the error do not have to be stored, because the summation can be calculated recursively according to the equation

$$S_N = \sum_{i=1}^N E_i = E_N + S_{N-1} \quad (11.5)$$

where  $S_{N-1} = \sum_{i=1}^{N-1} E_i$  and is stored from the previous controller execution.

The derivative is approximated by a backward difference. This approximation provides some smoothing; for example, the derivative of a perfect step is not infinite using equation (11.4), since  $\Delta t$  is never zero.

The three modes are combined into the full-position PID control algorithm:

$$\text{Full-position Digital PID} \quad MV_N = K_c \left[ E_N + \frac{\Delta t}{T_I} \sum_{i=1}^N E_i - \frac{T_d}{\Delta t} (CV_N - CV_{N-1}) \right] + I \quad (11.6)$$

Note that the constant of initialization is retained so that the manipulated variable does not change when the controller initiates its calculations.

Equation (11.6) is referred to as the *full-position algorithm* because it calculates the value to be output to the manipulated variable at each execution. An alternative approach would be to calculate only the change in the controller output at each execution, which is achieved with the *velocity* form of the digital PID:

$$\Delta MV_N = K_c \left[ E_N - E_{N-1} + \frac{\Delta t}{T_i} E_N - \frac{T_d}{\Delta t} (CV_N - 2CV_{N-1} + CV_{N-2}) \right] \quad (11.7)$$

Velocity  
Digital PID

$$MV_N = MV_{N-1} + \Delta MV_N \quad (11.8)$$

This equation is derived by subtracting the full-position equation (11.6) at sample  $N - 1$  from the equation at sample  $N$ . Either the full-position or the velocity form can be used, and many commercial systems are in operation with each basic algorithm.

The digital PID controller, either equation (11.6) or (11.7), can be rapidly executed in a process control computer. Only a few multiplications of current or recent past values times parameters and a summation are required. Also, little data storage is required for the parameters and few past values.

In conclusion, simple numerical methods are adequate for approximating the integral and derivative terms in the PID controller. As a result, the controller modes, set point, and tuning constants are the same in the digital PID algorithm as they are in the continuous algorithm. This is very helpful, because we can apply what we have learned in previous chapters about how the modes affect stability and performance to the digital algorithm. For example, it can be shown for the digital controller that the integral mode is required for zero steady-state offset and that the derivative mode amplifies high-frequency noise.

## 11.5 ■ EFFECTS OF DIGITAL CONTROL ON STABILITY, TUNING, AND PERFORMANCE

The tuning of continuous control systems is presented in Chapter 9, and stability analysis is presented in Chapter 10. A similar, mathematically rigorous analysis of the stability of digital control systems can be performed and is presented in Appendix L. This section provides the essential results without detailed mathematical proofs. The major differences in digital systems are highlighted, modifications to existing tuning guidelines are provided, and examples are presented to demonstrate the results. The measures of control performance and the definition of stability are the same as introduced in previous chapters.

As described in Section 11.3, sampling introduces an additional delay in the feedback system, and this delay is similar to, but not the same as, a dead time. Thus, we expect that longer sampling will tend to destabilize a feedback system and degrade its performance.

### EXAMPLE 11.2.

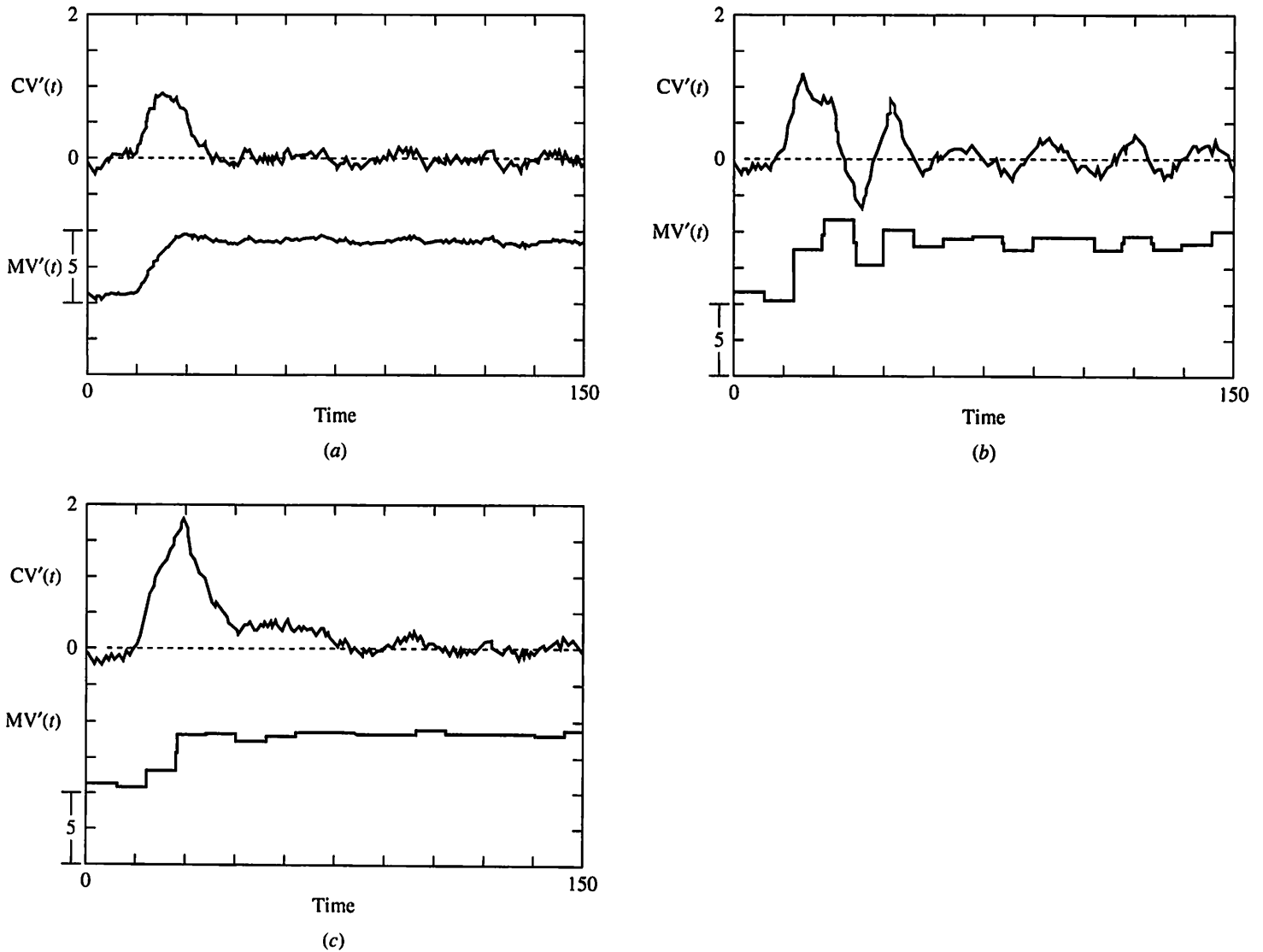
As an example, we consider a feedback control system for which the transfer functions for the process and disturbance are as follows and the disturbance is a

step of magnitude 3.6:

$$G_p(s) = \frac{-1.0e^{-2s}}{(10s+1)(0.2s+1)} \approx \frac{-1.0e^{-2.2s}}{(10s+1)} \quad (11.9)$$

$$G_d(s) = \frac{1}{(5s+1)(10s+1)} \quad D(s) = \frac{3.6}{s}$$

The performance of the system under continuous PI feedback control is given in Figure 11.7a using the Ciancone tuning from Figure 9.9. The performance is given in Figure 11.7b under discrete PI control with an execution period of 9, using the *same tuning* as in Figure 11.7a. We notice that the discrete response is more oscillatory and gives generally poorer performance. Several other responses were simulated, and their results are summarized in Table 11.2. When the execution period was made long, in this case 10 or greater, the control system became unstable!



**FIGURE 11.7**

Example process: (a) under continuous control; (b) under digital PI control with  $\Delta t = 9$  using continuous tuning; (c) under digital control with  $\Delta t = 9$  using altered tuning from Table 11.3.

TABLE 11.2

**Example of the performance of PI controllers for various execution periods with  $K_c = -1.7$  and  $T_I = 5.5$**

Execution period	IAE
Continuous	18.9
0.5	18.9
1.0	19.1
3.0	19.9
7.0	25.8
9.0	32.0
10.0	Unstable

This example shows that the control performance generally degrades for increasing sample periods and that the system can become unstable at long periods.

Since sampling introduces a delay in the feedback loop, we would expect that the tuning should be altered for digital systems to account for the sampling. The result in Section 11.3 indicated that the sample introduced an additional delay, which can be approximated as a dead time of  $\Delta t/2$ . Thus, one common approach for tuning PID feedback controllers and estimating their performance is to add  $\Delta t/2$  to the feedback process dead time and use methods and guidelines for continuous systems (Franklin, Powell, and Workman, 1990). The tuning rules developed in Chapters 9 and 10 can be applied to digital systems with the dead time used in the calculations equal to the process dead time plus one-half of the sample period (i.e.,  $\theta' = \theta + \Delta t/2$ ).

As demonstrated in Example 11.2, slow digital PID controller execution can degrade feedback performance. Also, as the execution becomes faster, i.e., as the execution period becomes smaller, the performance is expected to approach that achieved using a continuous controller. Thus, a key question is "Below what value of the PID controller execution period do the digital and continuous controllers provide nearly the same performance?" The following guideline, based on experience, is recommended for selecting controller execution period:

To achieve digital control performance close to continuous performance, select the PID controller execution period  $\Delta t \leq 0.05(\theta + \tau)$ , with  $\theta$  and  $\tau$  the feedback dead time and dominant time constant, respectively.

The execution period is proportional to the feedback dynamics, which seems logical because faster processes would benefit from faster controller execution. Many of the modern digital control systems have execution periods less than 1 sec; therefore, this guideline is easily achieved for most chemical processes. However, it may not be easily achieved for (1) fast processes, such as pressure control of

liquid-filled systems, or (2) control systems using analyzers that provide a new measurement infrequently.

The preceding discussion is summarized in the following tuning procedure:

1. Obtain an empirical model.
2. Determine the sample period [e.g.,  $\Delta t \leq 0.05(\theta + \tau)$ ].
3. Determine the tuning constants using appropriate methods (e.g., Ciancone correlation or Ziegler-Nichols method) with  $\theta' = \theta + \Delta t/2$ .
4. Implement the initial tuning constants and fine-tune.

As stated previously, the values obtained from these guidelines should be considered initial estimates of the tuning constants, which are to be evaluated and improved based on empirical performance through fine-tuning.

**EXAMPLE 11.3.**

Apply the recommended method to tune a digital controller for the process defined in equation (11.9). Results for several execution periods are given in Table 11.3, with the tuning again from the Ciancone correlations in Figure 9.9, and the control performance is shown in Figure 11.7c for an execution period of 9. Note that in all cases, including that with an execution period of 10, the dynamic performance is stable and well behaved (not too oscillatory). Recall that the performance could be improved (IAE reduced) with some fine-tuning, but at the expense of robustness.

It is apparent that the dynamic response is well behaved, with a reasonable damping ratio and moderate adjustments in the manipulated variable, when the digital controller is properly tuned. Also, it is clear that the performance of the digital controller is not as good as that of the continuous controller. In fact,

The performance of a continuous process under digital PID control is nearly always worse than under continuous control. The difference depends on the length of the execution period relative to the feedback dynamics.

**TABLE 11.3**

**Example of the performance of PI controllers for various execution periods with tuning adjusted accordingly**

Execution period $\Delta t$	Dead time $\theta' = \theta + \Delta t/2$	Fraction dead time $\theta'/(\theta' + \tau)$	$K_c$	$T_I$	IAE
Continuous	2.2	0.18	-1.7	5.5	18.9
1	2.7	0.21	-1.76	6.1	19.2
3	3.7	0.27	-1.50	8.9	26.8
5	4.7	0.32	-1.23	10.3	36.0
7	5.2	0.34	-1.2	10.6	37.2
9	6.7	0.40	-1.05	11.0	42.7
10	7.2	0.42	-1.0	11.1	44.8

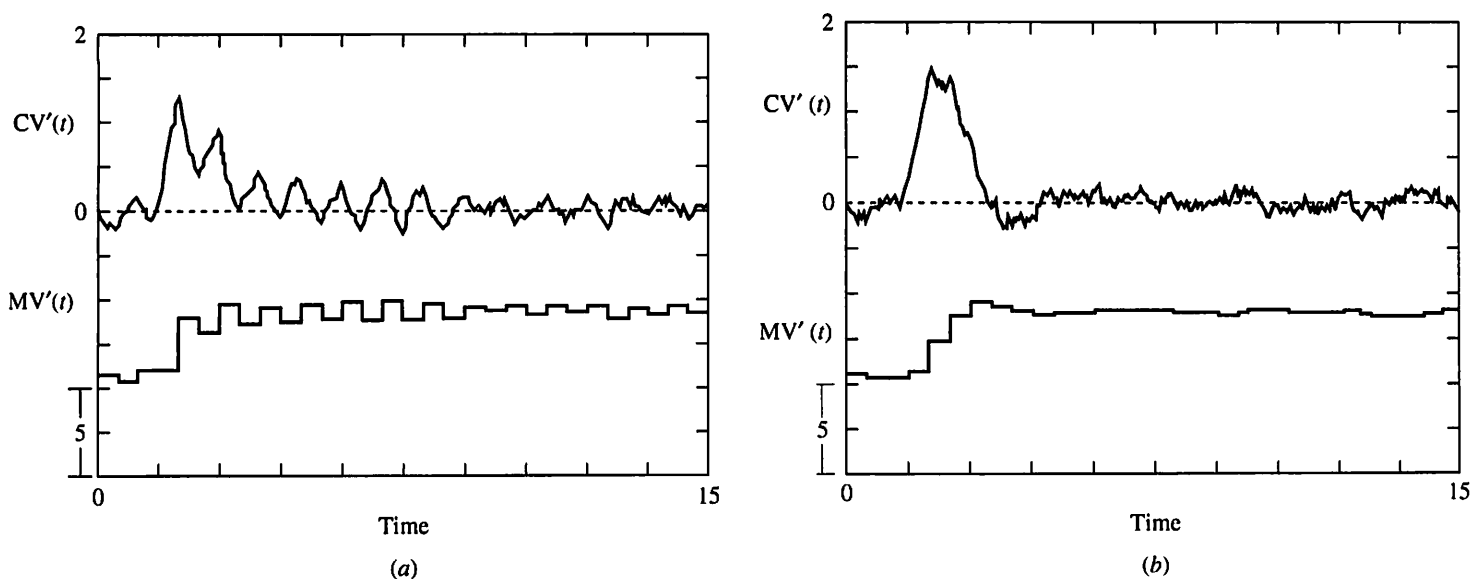
Note that the execution period is related to the dynamics of the feedback process, since "fast" and "slow" must be relative to the process. A guideline drawn from Table 11.3 is that the effect of sampling and digital control is not usually significant when the sample period is less than about  $0.05(\theta + \tau)$ . For a summary of many other guidelines, see Seborg, Edgar, and Mellichamp (1989).

Occasionally, controllers give poor performance that is a direct result of the digital implementation. This type of performance is shown in Figure 11.8a, in which a digital PI controller with a relatively slow execution period is controlling a process with very fast dynamics. The models for the process and controller are as follows:

$$\text{Process: } \frac{-1.0}{0.5s + 1} \quad (11.10)$$

$$\text{Controller: } MV_N = K_c \left( E_N + \frac{\Delta t}{T_I} \sum_{i=1}^N E_i \right) \quad \text{with } \Delta t = 0.5 \quad (11.11)$$

The oscillations in the manipulated variable are known as *ringing*. Diagnosing the causes of ringing requires mathematics ( $z$ -transforms) (Appendix L). However, the cause of this poor performance can be understood by considering the digital controller equation (11.11). The controller adjusts the manipulated variable to correct an error (e.g., a large positive adjustment). If a large percentage of the effect of the correction appears in the measured control variable at the next execution, the current error  $E_N$  can be small while the past error,  $E_{N-1}$ , will be large with a negative sign, causing a large negative adjustment in the manipulated variable. The result is an oscillation in the manipulated variable every execution period,



**FIGURE 11.8**

Digital control of a fast process (a) with  $K_c = -1.4$  and  $T_I = 7.0$ ; (b) with  $K_c = -0.14$  and  $T_I = 0.64$ .

which is very undesirable. In this case, the oscillations can be reduced by decreasing the controller gain and decreasing the integral time, so that the controller behaves more like an integral controller. The improved performance for the altered tuning is given in Figure 11.8*b*. This type of correction is usually sufficient to reduce ringing for PID control.

We have seen that the digital controller generally gives poorer performance than the equivalent continuous controller, although the difference is not significant if the controller execution is fast with respect to the feedback process.

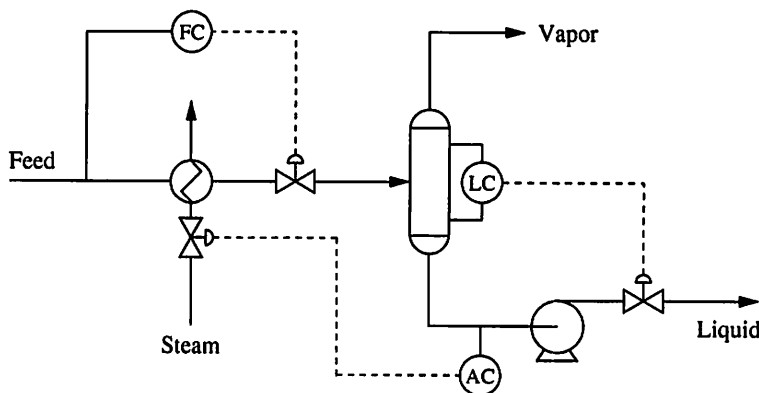
## 11.6 ■ EXAMPLE OF DIGITAL CONTROL STRATEGY

To demonstrate the analysis of a control system for digital control, the execution periods for the flash system in Figure 11.9 are estimated. The process associated with the flow controller is very fast; thus, the execution period should be fast, and perhaps, the controller gain may have to be decreased due to ringing. The level inventory would normally have a holdup time (volume/flow) of about 5 min, so that very frequent level controller execution is not necessary.

Let us assume that the analyzer periodically takes a sample from the liquid product and determines the composition by chromatography. In this case, the analyzer provides new information to the controller at the completion of each batch analysis, which can be automated at a period depending on the difficulty of separation. For example, a simple chromatograph might be able to send an updated measured value of the controlled variable every 2 min. Since the analyzer controller should be executed only when a new measured value is available, the controller execution period should be 2 min.

The execution periods can be approximated using the guideline of  $\Delta t = 0.05(\theta + \tau)$  for PID controllers, with the process parameters determined by one of the empirical model identification methods described in Chapter 6. Modern digital controllers typically execute most loops very frequently, usually with a period under 1 sec, unless the engineer specifies a longer period. The results for the example are summarized in Table 11.4.

Notice that the conventional digital systems might not satisfy the guideline for very fast processes, but the resulting small degradation of the control perfor-



**FIGURE 11.9**

Example process for selecting controller execution periods.



TABLE 11.4

**Digital controller execution periods for the example in Figure 11.9**

<b>Controlled variable</b>	<b>Maximum execution period</b>	<b>Typical execution period in commercial equipment</b>
Flow	0.2 sec	0.1 to 0.5 sec
Level	15 sec	0.1 to 0.5 sec
Analyzer	2 min	2 min

mance is not usually significant for most flows and pressures. When the variable is extremely important, as is the case in compressor surge control, which prevents damage to expensive mechanical equipment, digital equipment with faster execution (and sensors and valves with faster responses) should be used (e.g., Staroselsky and Ladin, 1979).

The analyzer has a very long execution period; therefore, it would be best to select the execution immediately when the new measured value becomes available, rather than initiate execution every 2 min whether the updated measurement is available or not. Thus, it is common practice for the controller execution to be synchronized with the update of a sensor with a long sample period; this is achieved through a special signal that indicates that the new measured value has just arrived.

## 11.7 □ TRENDS IN DIGITAL CONTROL

The basic principles presented in this chapter should not change as digital control equipment evolves. However, many of the descriptions of the equipment will undoubtedly change; in fact, the simple descriptions here do not attempt to cover all of the newer features being used. A few of the more important trends in digital control are presented in this section.

### Signal Transmission

The equipment described in this chapter involves analog signal transmission between the central control room and the sensor and valve. It is possible to collect a large number of signals at the process equipment and transmit the information via a digital communication line. This digital communication would eliminate many—up to thousands—of the cables and terminations and result in great cost savings. The reliability of this digital system might not be as good, because the failure of the single transmission line would cause a large number of control loops to fail simultaneously. However, the potential economic benefit provides a driving force for improved, high-reliability designs. This is a rapidly changing area for which important standards are being developed that should facilitate the integration of equipment from various suppliers (Lidner, 1990; Thomas, 1999).

It is possible to communicate without physical connections, via *telemetry*. This method is now used to collect data from remote process equipment such as crude petroleum production equipment over hundreds of kilometers. When telemetry is sufficiently reliable, some control could be implemented using this communication method.

## **Smart Sensors**

Microprocessor technology can be applied directly at the sensor and transmitter to provide better performance. An important feature of these sensors is the ability for *self-calibration*—that is, automatic corrections for environmental changes, such as temperature, electrical noise, and process conditions.

## **Operator Displays**

Excellent displays are essential so that operating personnel can quickly analyze and respond to ever-changing plant conditions. Current displays consist of multiple cathode ray tubes (CRTs). Future display technology is expected to provide flat screens of much larger area. These larger screens will allow more information to be displayed concurrently, thus improving process monitoring.

## **Controller Algorithms**

The flexibility of digital calculations eliminates a restriction previously imposed by analog computation that prevented engineers from employing complex algorithms for special-purpose applications. Some of the most successful new algorithms use explicit dynamic models in the controller. These algorithms are presented in this book in Chapter 19 on predictive control and in Chapter 23 on centralized multivariable control.

## **Monitoring and Optimization**

The large amount of data collected and stored by digital control systems provides an excellent resource for engineering analysis of process performance. The results of this analysis can be used to adjust the operating conditions to improve product quality and profit. This topic is addressed in Chapter 26.

### **11.8 ■ CONCLUSIONS**

Digital computers have become the standard equipment for implementing process control calculations. However, the trend toward digital control is not based on better performance of PID control loops. In fact, the material in this chapter demonstrates that most PID control loops with digital controllers do not perform as well as those with continuous controllers, although the difference is usually very small.

The sampling of a continuous measured signal for use in feedback control introduces a limit to control performance, because some high-frequency information is lost through sampling. Shannon's theorem provides a quantitative estimate of the frequency range over which information is lost.

Sampling and discrete execution introduce an additional dynamic effect in the feedback loop, which influences stability and performance. Guidelines are provided that indicate how the PID controller tuning should be modified to retain the proper margin from the stability limit while providing reasonable control performance. As we recall, the stability margin is desired so that the control system performs well when the process dynamic response changes from its estimated value—in other words, so that the system performance is robust.

A major conclusion from this chapter is that

The characteristics of the modes and tuning constants for the continuous PID controller can be interpreted in the same manner for the digital PID controller. The digital PID controller must use modified tuning guidelines to achieve good performance and robustness.

This valuable result enables us to apply the same basic concepts to both continuous analog and digital controllers.

The power of digital computers is in their flexibility to execute other control algorithms easily, even if the computations are complex.

All control methods described in subsequent chapters can be implemented in either analog or digital calculating equipment, unless otherwise stated. Where the digital implementation is not obvious, the digital form of the controller algorithm is given.

This power will be capitalized on when applying advanced methods such as non-linear control (Chapters 16 and 18), inferential control (Chapter 17), predictive control (Chapters 19 and 23), and optimization and statistical monitoring (Chapter 26).

Many of the guidelines and recommendations in this chapter have been verified through simulation examples. For continuous control systems, rigorous proofs and methods of analysis have been provided using Laplace transforms, for example, in Chapter 10 (and the forthcoming Chapter 13). Similar analysis methods are available for digital control systems using  $z$ -transforms. An introduction to  $z$ -transforms and their application to digital control systems analysis are provided in Appendix L.

## REFERENCES

- Astrom, K., and B. Wittenmark, *Computer Controlled Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1990.
- Franklin, G., J. Powell, and M. Workman, *Digital Control of Dynamic Systems* (2nd ed.), Addison-Wesley, Reading, MA, 1990.
- Gardenhire, L., "Selecting Sampling Rates," *ISA J.*, 59–64 (April 1964).
- Gerald, C., and P. Wheatley, *Applied Numerical Analysis* (4th ed.), Addison-Wesley, Reading, MA, 1989.
- Lidner, K-P., "Fieldbus—A Milestone in Field Instrumentation Technology," *Meas. Control*, 23, 272–277 (1990).
- Seborg, D., T. Edgar, and D. Mellichamp, *Process Dynamics and Control*, Wiley, New York, 1989.
- Staroselsky, N., and L. Ladin, "Improved Surge Control for Centrifugal Compressors," *Chem. Engr.*, 86, 175–184 (May 21, 1979).
- Thomas, J., "Fieldbuses and Interoperability," *Control Engineering Practice*, 7, 81–94 (1999).

## ADDITIONAL RESOURCES

Each commercial digital control system has an enormous array of features, making comparisons difficult. A summary of the equipment for some major suppliers is provided in the manual

Wade, H. (ed.), *Distributed Control Systems Manual*, Instrument Society of America, Research Triangle Park, NC, 1992 (with periodic updates).

In addition to the references by Astrom and Wittenmark (1990) and by Franklin et al. (1990), the following book gives detailed information on z-transforms and digital control theory.

Ogata, K., *Discrete-Time Control Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

For an analysis of digital controller execution periods that considers the disturbance dynamics for statistical, rather than deterministic, disturbances, see

MacGregor, J., "Optimal Choice of the Sampling Interval for Discrete Process Control," *Technometrics*, 18, 2, 151–160 (May 1976).

## QUESTIONS

**11.1.** Answer these questions about the digital PID algorithm.

- (a) Give the equations for the full-position and velocity PID controllers if a trapezoidal numerical integration were used for the integral mode.
- (b) The digital controller can be simplified to the following form to reduce real-time computations. Determine the values for the constants (the  $A_i$ 's) in terms of the tuning constants and execution period for (i) PID, (ii) PI, and (iii) PD controllers.

$$\Delta MV_N = A_1 E_N + A_2 E_{N-1} + A_3 CV_N + A_4 CV_{N-1} + A_5 CV_{N-2}$$

**11.2.** Many tuning rules were designed for continuous control systems, such as Ziegler-Nichols, Ciancone, and Lopez.

- (a) Describe the conditions, including quantitative measures, for which these tuning rules could be applied to digital controllers without modification.
- (b) How could you adjust the rules to systems that had longer execution periods than determined by the approximate guidelines given in part (a) of this question?

**11.3.** Develop a simulation of a simple process under digital PID control. Equations for the process are given below. The calculations can be performed using a spreadsheet or a programming language. The input change is a step set point change from 1 to 2.0 at time = 1.0. The process parameters can be taken from the system in Section 9.3;  $K_p = 1.0$ ,  $\tau = 5.0$ , and  $\theta = 5.0$ , and the controller and simulation time steps can be taken to be equal; that

is,  $\delta t = \Delta t = 1.0$ .

$$\text{Process: } CV_N = (e^{-\delta t/\tau})CV_{N-1} + K_p(1 - e^{-\delta t/\tau})MV_{N-\Gamma-1} \quad \Gamma = \frac{\theta}{\delta t}$$

$$\text{Controller: } MV_N = MV_{N-1} + K_c \left( E_N - E_{N-1} + \frac{\Delta t}{T_I} E_N \right)$$

with  $\delta t =$  step size for the numerical solution of the process equation

$\Delta t =$  execution period of the digital controller

- Verify the equations for the process and controller and determine the initial conditions for MV and CV.
- Repeat the study summarized in Figure 9.2 for a set point change.
- Use the tuning in Table 9.2 to obtain the IAE for set point changes.
- Select tuning from several points on the response surface in Figure 9.3. Obtain the dynamic responses and explain the behavior: oscillatory, overdamped, and so forth.

**11.4.** Repeat question 11.3 for the system in Example 8.5 and obtain the dynamic response given in Figure 8.9. You must determine all parameters in the equations, including appropriate values for the process simulation step size and the execution period of the digital controller. Solve this problem by simulating (1) the linearized process model and (2) the nonlinear process model.

**11.5.** State for each of the controller variables in the following list

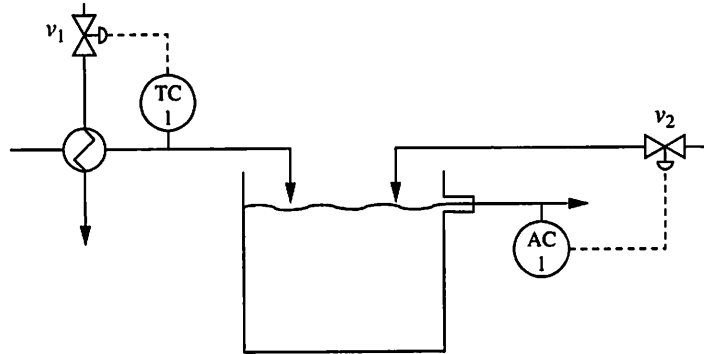
- its source (e.g., from an operator, from process, or from a calculation)
- whether the variable would be transferred to the operator console for display
  - $SP_N$ , the controller set point
  - $CV_N$ , the current value of the controlled variable
  - $K_c$ , the controller gain
  - $S_N$ , the sum of all past errors used in approximating the integral error
  - $MV_N$ , the current controller output
  - M/A, the status of the controller (M=manual or off, A=automatic or on)
  - $\Delta MV_N$ , the current change to the manipulated variable
  - $E_N$ , the current value of the error

**11.6.** A process control design is given in Figure Q11.6. The process transfer functions  $G_p(s)$  follow, with time in minutes:

$$(G_p(s))_T = \frac{T_1(s)}{v_1(s)} = \frac{3e^{-1.2s}}{1 + 2s} \quad \left( \frac{^\circ\text{C}}{\% \text{ open}} \right)$$

$$(G_p(s))_A = \frac{A_1(s)}{v_2(s)} = \frac{1.3e^{-0.5s}}{1 + 14s} \quad \left( \frac{\text{wt}\%}{\% \text{ open}} \right)$$

- For each controller, determine the maximum execution period so that digital execution does not significantly affect the control performance.
- Determine the PID controller tuning for each controller for two values of the execution period:
  - The result in (a) and (2) a value of 3 minutes


**FIGURE Q11.6**

- 11.7.** In the chapter it was stated that the digital controller should not be executed faster than the measured controlled variable is updated. In your own words, explain the effect of executing the controller faster than the measurement update and why this effect is undesirable.
- 11.8.** An example of ringing occurs when a digital proportional-only controller is applied to a process that is so fast that it reaches steady state within one execution period,  $\Delta t$ . The following calculations, which are simple enough to be carried out by hand, will help explain ringing.
- Calculate several steps of the response of a control system with a steady-state process with  $K_p = 1.0$  ( $\theta = \tau = 0$ ) and a proportional-only controller,  $K_c = 0.8$ . Assume that the system is initially at steady state and a set point change of 5 units is made.
  - Repeat the calculation for an integral-only controller, equation (8.16). Find a value of the parameter ( $K_c \Delta t / T_I$ ) by trial and error that gives good dynamic performance for the controlled and manipulated variables.
  - Generalize the results in (a) and (b) and give a tuning rule for integral-only, digital control of a fast process.
- 11.9.** Some example process dynamics and associated digital feedback execution periods are given in the following table. For each, calculate the PI controller tuning constants, assuming standard control performance objectives.

	<b>Process transfer function</b> $G_p(s)$	<b>Execution period</b> $\Delta t$
(a)	Three-tank mixer, Example 7.2	Selected by reader
(b)	Recycle system in equation (5.51)	Selected by reader
(c)	$\frac{1.2e^{-0.1s}}{1 + 0.5s}$	0.25
(d)	$\frac{1.2e^{-0.1s}}{1 + 0.5s}$	5.0
(e)	$\frac{2.1e^{-20s}}{1 + 100s}$	30
(f)	$\frac{2.1e^{-100s}}{1 + 20s}$	30

- 11.10.** Considering the description of a distributed digital control system, determine which processors, signal converters, and transmission equipment must act and in what order for (a) the result of an operator-entered set point change to reach the valve; (b) a process change to be detected and acted upon by the controller so that the valve is adjusted.
- 11.11.** Consider a signal that is a perfect sine with period  $T_{\text{signal}}$ , and is sampled at period  $T_{\text{sample}}$ , with  $T_{\text{sample}} < T_{\text{signal}}$ . Determine the primary aliasing frequency (the sample frequency at which the sampled values are periodic with a period a multiple of the true signal sine frequency) as a function of the two periods.
- 11.12.** (a) Determine bounds on the error between the continuous signal and the output of the sample/hold for a zero-order and a first-order hold. (Hint: Consider the rate of change of the continuous signal.)  
(b) Apply the results in (a) to a continuous sine signal and determine the errors for various values of the sample period to sine period.  
(c) Which hold gives a smaller error in (b)?
- 11.13.** Answer the following questions regarding the computer implementation of the digital PID controller.  
(a) Can the controller tuning constants be changed while the controller is functioning without disturbing the manipulated variable? (Consider the velocity and full-positional forms separately.)  
(b) For the velocity form of the PID, what is the value for  $MV_{N-1}$  for the first execution of the controller?  
(c) For the full-position form of the PID, the sum of the error term might become very large and overflow the word length. Is this a problem likely to occur?  
(d) Discuss how the calculations could be programmed to introduce limits on the change of the manipulated variable ( $\Delta MV_N$ ), the set point ( $SP_N$ ), and the manipulated variable ( $MV_N$ ).  
(e) Can you anticipate any performance difficulties when the limitations in (d) are implemented? If yes, suggest modifications to the algorithm.