PID Controller Tuning for Dynamic Performance

CHAPTER 9

9.1 INTRODUCTION

As demonstrated in the previous chapter, the proportional-integral-derivative (PID) control algorithm has features that make it appropriate for use in feedback control. Its three adjustable tuning constants enable the engineer, through judicious selection of their values, to tailor the algorithm to a wide range of process applications. Previous examples showed that good control performance can be achieved with a proper choice of tuning constant values, but poor performance and even instability can result from a poor choice of values. Many methods can be used to determine the tuning constant values. In this chapter a method is presented that is based on the time-domain performance of the control system. Controller tuning methods based on dynamic performance have been used for many decades (e.g., Lopez et al., 1969; Fertik, 1975; Zumwalt, 1981), and the method presented here builds on these previous studies and has the following features:

1. It clearly defines and applies important performance issues that must be considered in controller tuning.
2. It provides easy-to-use correlations that are applicable to many controller tuning cases.
3. It provides a general calculation approach applicable to nearly any control tuning problem, which is important when the general correlations are not applicable.

4. It provides insight into important relationships between process dynamic model parameters and controller tuning constants.

9.2 DEFINING THE TUNING PROBLEM

The entire control problem must be completely defined before the tuning constants can be determined and control performance evaluated. Naturally, the physical process is a key element of the system that must be defined. To consider the most typical class of processes, a first-order-with-dead-time plant model is selected here because this model can adequately approximate the dynamics of processes with monotonic responses to a step input, as shown in Chapter 6. Also, the controller algorithm must be defined; the form of the PID controller used here is

$$\mathrm{MV}(t) = K_c\left[E(t) + \frac{1}{T_I}\int_0^t E(t')\,dt' - T_d\frac{d\mathrm{CV}(t)}{dt}\right] + I \tag{9.1}$$

Note that the derivative term is calculated using the measured controlled variable, not the error.

> **The tuning constants must be derived using the *same* algorithm that is applied in the control system. The reader is cautioned to check the form of the PID controller algorithm used in developing tuning correlations and in the control system computation; these must be compatible.**

Next, we carefully define control performance by specifying several goals to be balanced concurrently. This definition provides a comprehensive specification of control performance that is flexible enough to represent most situations. The three goals are the following:

1. *Controlled-variable performance.* The well-tuned controller should provide satisfactory performance for one or more measures of the behavior of the controlled variable. As an example, we shall select to minimize the IAE of the controlled variable. The meaning of the integral of the absolute value of the error, IAE, is repeated here.

$$\mathrm{IAE} = \int_0^\infty |\mathrm{SP}(t) - \mathrm{CV}(t)|\,dt \tag{9.2}$$

   Zero steady-state offset for a steplike system input is ensured by the integral mode appearing in the controller.
2. *Model error.* Linear dynamic models always have errors, because the plant is nonlinear and its operation changes. Since the tuning will be based on these models, the tuning procedure should account for the errors, so that acceptable control performance is provided as the process dynamics change. The changes are defined as ± percentage changes from the base-case or nominal model parameters. The ability of a control system to provide good performance when the plant dynamics change is often termed *robustness*.
3. *Manipulated-variable behavior.* The most important variable, other than the controlled variable, is the manipulated variable. We shall choose the com-

TABLE 9.1

Summary of factors that must be defined in tuning a controller

Major loop component	Key factor	Values used in this chapter for examples and correlations
Process	Model structure	Linear, first-order with dead time
	Model error	± 25% in model parameters (structured so that all parameters increase and decrease the same %)
	Input forcing	Step input disturbance with $G_d(s) = G_p(s)$ and step set point considered separately
	Measured variable	Unbiased controlled variable with high-frequency noise
Controller	Structure	PID and PI
	Tuning constants	K_c, T_I, and T_d
Control performance	Controlled-variable behavior	Minimize the total IAE for several cases spanning a range of plant model parameter errors
	Manipulated-variable behavior	Manipulated variable must not have variation outside defined limits; see Figure 9.4

mon goal of preventing "excessive" variation in the manipulated variable by defining limits on its allowed variation, as explained shortly.

To evaluate the control performance, the goals and the scenario(s) under which the controller operates need to be defined. These definitions are summarized in Table 9.1; the general factors are in the second column, and the specific values used to develop correlations in this chapter are in the third column. This may seem like a rather lengthy list of factors to establish before tuning a controller, but they are essential to any proper tuning method. Fortunately, the rather standard set of specifications in the third column is appropriate for a wide range of applications, and therefore it is possible to develop correlations that can be used in many plants, *where this underlying specification of control performance is valid.* The entries in Table 9.1 will be further explained as they are encountered in the next section. All subsequent chapters in this book require a good understanding of the factors that affect control performance.

> The reader is encouraged to understand the factors in Table 9.1 thoroughly and to refer back to this section often when covering later chapters.

9.3 ▫ DETERMINING GOOD TUNING CONSTANT VALUES

Given a complete definition of the process, controller, and control objectives, evaluating the tuning constants is a relatively straightforward task, at least conceptually. The "best" tuning constants are those values that satisfy the control performance goals. With our definitions of Goals 1 to 3, the optimum tuning gives the minimum IAE, for the selected plant (with variations in model parameters), when the manipulated variable observes specified bounds on its dynamic behavior.

The control objectives in Table 9.1 have been defined so that they can be quantitatively evaluated from the dynamic response of a control system. The dynamic response of the control system with a complex process model including dead time cannot be determined analytically, but it can be evaluated using a numerical solution of the process and controller equations. The dynamic equations are solved from the initial steady state to the time at which the system attains steady state after the input change. The best values of the tuning constant can be determined by evaluating many values and selecting the values that yield best measure of control performance. Since the goal of this presentation is to concentrate on the *effects of the process dynamics on tuning*, not the detailed mathematics, the reader may visualize the best values being found by a grid search over a range of the tuning constant values, although this procedure would involve excessive computations. (Some further details on the solution approach are given in Appendix E.) The result is a set of tuning (K_c, T_I, T_d) that gives the best performance for a specific plant, model uncertainty, and control performance definition.

As explained in Section 9.2, we will consider a first-order-with-dead-time plant because this model can (approximately) represent the dynamics of many overdamped processes. As a helpful image for the reader, a simple mixing process example shown in Figure 9.1 will be used throughout this chapter, although the results are not limited to this simple process, as will be demonstrated later in the chapter. The process can be described by the following transfer function model:

$$G_v(s)G'_p(s)G_s(s) = G_p(s) = \frac{K_p e^{-\theta s}}{\tau s + 1} \quad \text{(\%A in outlet)/(\%valve opening)} \tag{9.3}$$

$$G_d(s) = \frac{K_d}{\tau s + 1} \quad \text{(\%A in outlet)/(\%A in inlet)} \tag{9.4}$$

From a fundamental balance on component A, the dead time and time constant can be determined as the following functions of the feed flow rate and equipment size.

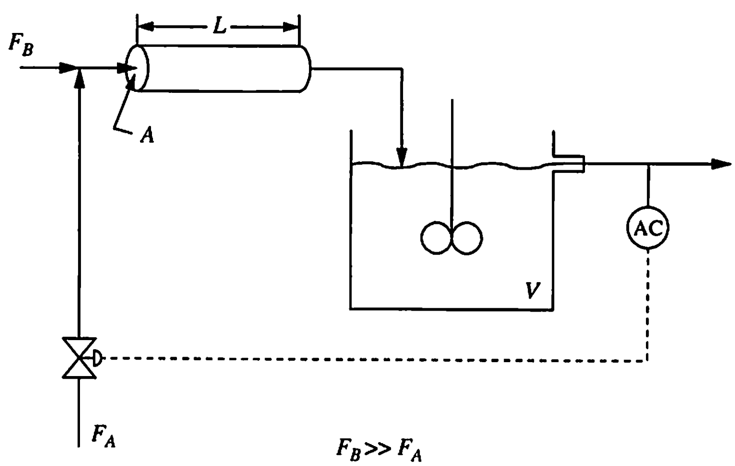


FIGURE 9.1

Process used for calculating example tuning constants for good control performance.

The base case values are given here, and the functional relationships will be used in later examples to determine the modified dynamics for changes in production rate (F_B).

Parameter	Dependence on process	Base case value
Dead time, θ	$(A)(L)/F_B$	5.0 min
Time constant, τ	V/F_B	5.0 min
Steady-state gain, K_P	$K_v[(x_A)_A - (x_A)_B]/F_B$	1.0 (%A in outlet)/(%open)

In general, the three tuning constants (K_c, T_I, and T_d) should be evaluated *simultaneously* to achieve the best performance. However, we will gain considerable insight by considering the PID tuning constants and performance goals sequentially. This will enable us to learn how the goals influence the values of the tuning constants and also the interaction among the values of the three tuning constants. Therefore, we shall begin with the simplest case, determining the value of one tuning constant, K_c, which results in the minimum in the performance measure goal 1 (IAE). In this initial case, the other two tuning constant values (T_I and T_d) will be held constant at reasonable values. Then, values of all three tuning constants will be determined that give the best control performance, as represented by goal 1 (IAE). Finally, the values of the tuning constants are determined that give the best performance, as measured by the complete definition of control performance, goals 1 to 3.

Recall that the feedback control system is designed to respond to disturbances and changes in set points (desired values). Initially, we will restrict attention to a unit step disturbance in the inlet concentration, $D(s) = 1/s$ %A in the inlet. Later, set point changes will be addressed.

Goal 1: Controlled-Variable Performance (IAE)

Let us begin with a PID controller applied to the example process. We will start by optimizing only one controller constant. Recall that the integral mode is required so that the controlled variable returns to its set point. Therefore, the study will find the best value of the controller gain, K_c, with the integral time ($T_I = 10$ min) and derivative time ($T_d = 0$ min) temporarily maintained at fixed values. The value selected for the integral time (the sum of the dead time and time constant) is reasonable (although not optimum), as demonstrated by further results, and the derivative time of zero simply turns off the derivative mode. For this first case, the goal in this analysis is temporarily limited to achieving the minimum value of the IAE for the base case plant model.

The results of several transient responses are presented in Figure 9.2, with each case having a different value of the controller gain. The results show that the relationship between IAE and K_c is *unimodal;* that is, it has a single minimum. The minimum IAE is at a controller gain value of about $K_c = 1.14\%/(\text{mole/m}^3)$ with an IAE of 9.1. For values of the controller gain smaller than the best value (e.g., $K_c = 0.62$), the controller is too "slow," leading to higher IAE. For values

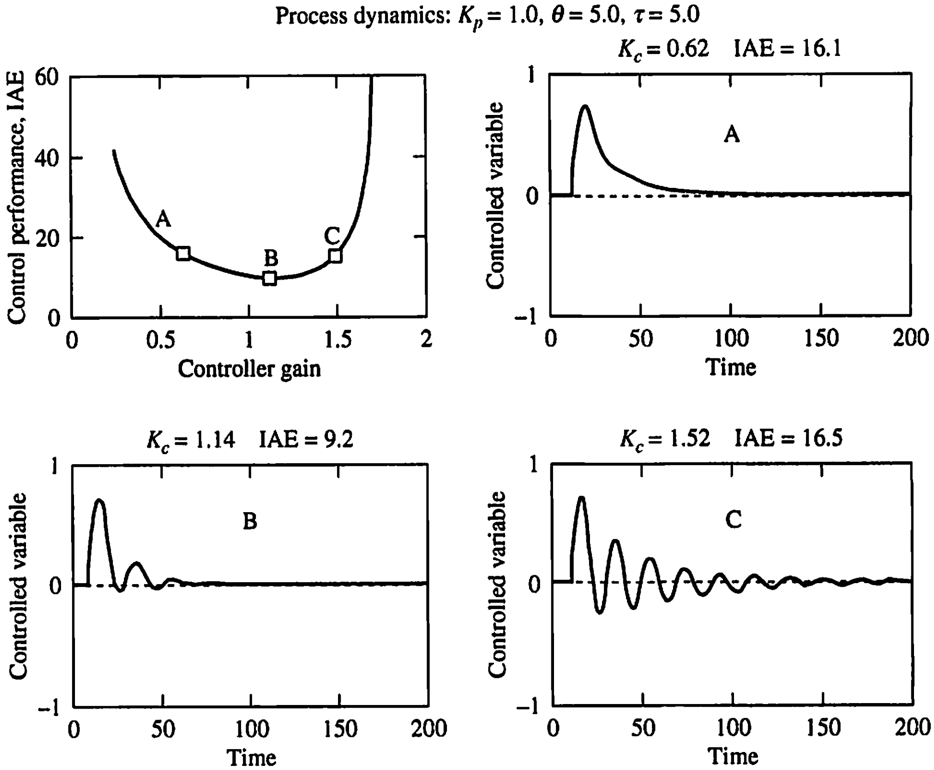


FIGURE 9.2

Dynamic responses used to determine the best controller gain, K_c% open/ %A, with $T_I = 10$ and $T_d = 0$.

of the controller gain larger than the best value (e.g., $K_c = 1.52$), the controller is too "aggressive," leading to oscillations and higher IAE. Note that the optimum is somewhat "flat"; that is, the control performance does not change very much for a range (about $\pm 15\%$) about the optimum controller gain. However, if the controller gain is increased too much, the system will become unstable. (Determining the stability limit is addressed in the next chapter.)

The graphical presentation used for one constant can be extended to two constants by varying the controller gain and integral time simultaneously while holding the derivative time constant ($T_d = 0$). Again, many dynamic responses can be evaluated and the results plotted. In this case, the coordinates are the controller gain and integral time, with the IAE plotted as contours. The results are presented in Figure 9.3, where the optimum tuning is $K_c = 0.89$ and $T_I = 7.0$. Again, the same qualitative behavior is obtained, with very large or small values of either constant giving poor control performance. In addition, the contours show the interaction between the variables; for example, nearly the same control performance can be achieved by gain and integral time values of ($K_c = 0.6$ and $T_I = 4.5$) and ($K_c = 1.2$ and $T_I = 10$), respectively. Again, the control performance is not too sensitive to the tuning values, as shown by the large region (valley) in which the performance changes by only about 10 percent. Finally, the evaluations identified a region in which the control system is not stable; that is, where the IAE becomes infinite. It is interesting that the region of good control performance—the lower valley in the contour plot—runs nearly parallel to the stability bound. This result will be used

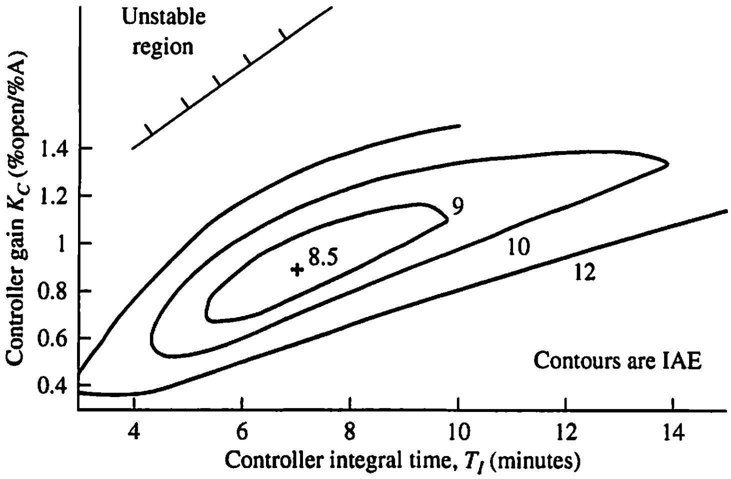


FIGURE 9.3

Contours of controller performance, IAE, for values of controller gain and integral time.

TABLE 9.2

Summary of tuning study

Case	Objective	Gain, K_c (%/%A)	Integral time, T_I (min)	Derivative time, T_d (min)	IAE+
Optimize K_c	Goal 1 (IAE)	1.14	10.0 (fixed)	0.0 (fixed)	9.2
Optimize K_c and T_I	Goal 1 (IAE)	0.89	7.0	0.0 (fixed)	8.5
Optimize K_c, T_I, and T_d	Goal 1 (IAE)	1.04	5.3	2.1	5.8
Optimize K_c, T_I, and T_d	Goal 1–3 simultaneously	0.88	6.4	0.82	7.4* ←recommended

+Evaluated for nominal model (without error) without noise. Process parameters were the gain $K_p = 1.0$%A/%, the time constant $\tau = 5$ minutes, and the dead time $\theta = 5$ minutes.
*Greater than 5.8 because of additional goals 2 and 3.

in the next chapter, in which the stability of control systems is studied and tuning constant values are determined based on a margin from the stability bound.

When three or more values are optimized, as is the case for a three-mode controller, the results cannot be displayed graphically. One could take the same optimization procedure described for one- and two-variable problems, which is simply to evaluate the IAE over a grid of tuning constant values and estimate the best values from the results or use a more sophisticated and efficient approach. The application of an optimization to the example process yields values of all three parameters that minimize IAE, and the values are reported in Table 9.2. This table summarizes the results with one, two, and all three constants being optimized; clearly, as more constants are free for adjustment, the IAE controller performance measure improves (i.e., decreases). Also, the optimum values for the controller gain and integral time change when we include the derivative time as an adjustable variable in the optimization. This result again demonstrates the *interaction* among the tuning constants.

Minimizing the IAE is only the first of the three specified goals, which considers the behavior of only the controlled variable and assumes perfect knowledge (model) of the process. This preliminary result does *not* provide the best control performance according to our specified goals; therefore, we must continue to refine the procedure to determine the best tuning constant values.

Goal 2: Good Control Performance with Model Errors

To this point we have determined tuning constant values that minimize the IAE when the process dynamics are described *exactly* by the base case dynamic model. However, the model is never perfect, because of errors in the model identification procedure, as demonstrated in Chapter 6. Also, plant operating conditions, such as production rate, feed composition, and purity level, change, and because processes are nonlinear, these changes affect the dynamic behavior of the feedback process. The effect of changing operating conditions can be estimated by evaluating the linearized models at different conditions and determining the changes in gain, time constant, and dead time from their base-case values. Since the true process dynamic behavior changes, a useful tuning procedure should determine tuning constants that give good performance for a range of process dynamics about the base case or nominal model parameters, as required by the second control performance goal. When the tuning results in satisfactory performance for a reasonable range of process dynamics, the tuning is said to provide *robustness*.

> In performing control and tuning analyses, the engineer must define the expected model error. The error estimate, usually expressed as ranges of parameters, can be based on the variation in plant operation and fundamental models from Chapters 3 through 5 or the results of several empirical model identifications using the methods in Chapter 6.

The size and type of model error is process-specific. For the purposes of developing correlations, the major source of variation in process dynamics is assumed to result from changes in the flow rate of the feed stream F_B in Figure 9.1 that cause $\pm 25\%$ changes in the parameters. While the range of parameters depends on the specific process, most processes experience parameter value changes of roughly this magnitude, and some have much larger variations. The resulting model parameters are given in Table 9.3; these values can be derived using the expressions already given relating the linearized model parameters to the process design and operation. Since in this example all parameters are proportional to the inverse of the feed flow, the parameters do not vary independently but in a *correlated* manner as a result of changes in input variables. Such correlation among parameter variation is typical, because the major cause of variation in process dynamics is nonlinearity. Naturally, the functional relationship depends on the process and is not always as shown in the table.

TABLE 9.3

Model parameters for the three-tank process

Model parameters	Low flow, $i = 1$	Base case flow, $i = 2$	High flow, $i = 3$
K_p	1.25	1.0	0.75
θ	6.25	5.0	3.75
τ	6.25	5.0	3.75

The goal is to provide good control performance for this range, and one way to consider the variability in dynamics is to modify the objective function to be the sum of the IAE for the three cases, which include the base case and the extremes of low and high flow rates in Table 9.3. The objective is stated as follows:

$$\text{Minimize} \qquad \sum_{i=1}^{3} \text{IAE}_i \tag{9.5}$$

$$\text{by adjusting} \qquad K_c, T_I, T_d$$

$$\text{IAE}_i = \int_0^\infty |\text{SP}(t) - \text{CV}_i(t)|dt$$

where $\text{CV}_i(t)$ is calculated using process parameters for $i = 1$ to 3 in Table 9.3.

This modification is very important, because tuning constants that yield good performance for the nominal model may give poor performance or even result in instability as the true process parameters vary. Next, the third goal is discussed; afterward, the tuning constants satisfying all three goals are determined.

Goal 3: Manipulated-Variable Behavior

The third and final goal addresses the dynamic behavior of the manipulated variable by requiring it to observe a limitation. As previously discussed, its variation should not be too great, because of wear to control and process equipment and disturbances to integrated units. There are many ways to define the variation of the manipulated variable. Here we will bound the allowed transient path of the manipulated variable to a specified region around the final steady-state value during the dynamic response as shown in Figure 9.4. This rather general limitation enables us to address two related issues in manipulated-variable variation:

1. The largest-magnitude variation in the manipulated variable in response to a disturbance or set point change
2. The high-frequency variation resulting from the small, continuous changes in the controlled variable often referred to as *noise*

The allowable manipulated-variable range is large during the initial part of the transient, where, in general, the manipulated variable should be able to overshoot its final value. The range is smaller after the effect of the step disturbance is corrected.

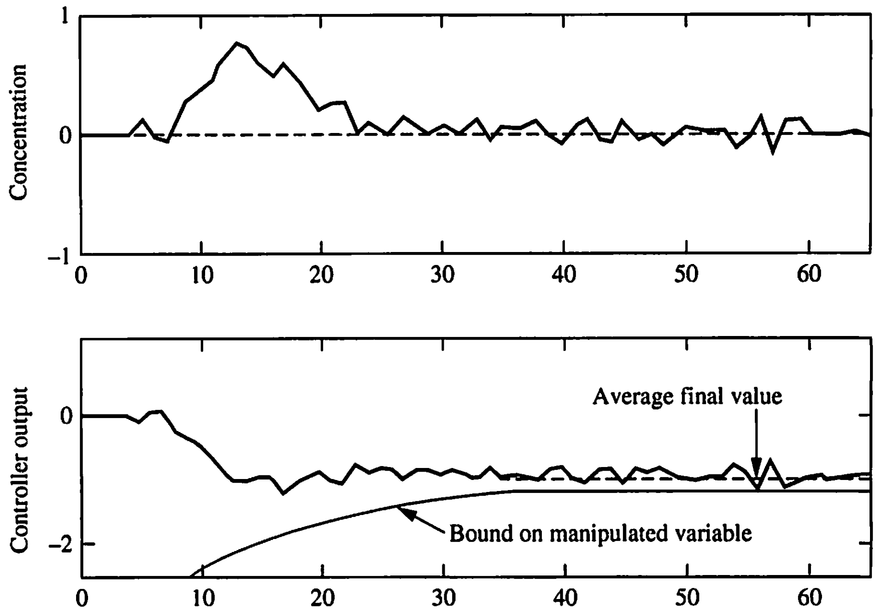


FIGURE 9.4

Dynamic response of a feedback control system showing the bound on allowable manipulated-variable adjustments.

Even after a long time, the manipulated variable cannot be required to be absolutely constant, because feedback control responds to the small, continuous changes in the controlled variable (i.e., the noise). The limitation on the manipulated variable is determined by parameters that define the bound shown in Figure 9.4. Simulations to evaluate a tuning for goals 1 through 3 include representative noise on the measured, controlled variable and a bound on the manipulated variable. A model for defining the bound on the path, along with parameters used in this book, is presented in Appendix E.

The proper values of the parameters used to define the allowed manipulated variable behavior should match the process application. The values in this study are good initial estimates for many process control designs. However, the specific parameter values are not the key concept in this goal statement; what is most important is this:

A properly defined statement of control performance includes a specification of acceptable manipulated-variable behavior.

Since both controlled- and manipulated-variable plots of behaviors are important, most closed-loop transient responses in this book show both the controlled and manipulated variables; in general, it is not possible to evaluate control performance by observing only the controlled variable.

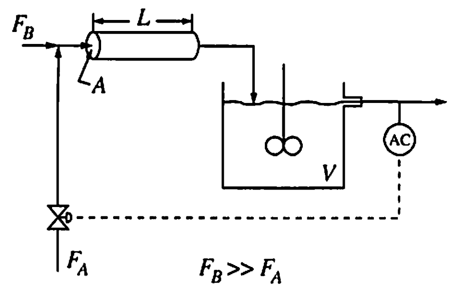
The controller constants in the example mixing process are optimized for the complete definition, and the results are $K_c = 0.88$, $T_I = 6.4$, and $T_d = 0.82$. The dynamic response is given in Figure 9.4 for the nominal plant response. (Recall that three dynamic responses, including model error, were considered concurrently in determining the optimum.) These tuning parameters satisfy goals 1 through 3 in our control performance definition. Note that compared to the results reported in Table 9.2, which satisfy only goal 1, the values satisfying all three goals have a lower gain, longer integral time, and shorter derivative time. Thus:

> The controller is *detuned*, leading to less aggressive adjustments by the feedback controller, to account for modelling errors and to reduce the variation in the manipulated variable.

These tuning constants will not perform best when the model error is zero and no noise is present, but they will perform better over an expected range of conditions and are the values recommended for initial application.

EXAMPLE 9.1.

A modified process in Figure 9.1, with a shorter pipe and larger tank described by the nominal model in equation (9.6), is to be controlled by a PID controller. Determine the best initial tuning constant values for a PID controller based on (*a*) goal 1 alone and (*b*) goals 1 through 3.



$$G_v(s)G'_p(s)G_s(s) \approx G_p(s) = \frac{1.0e^{-2s}}{8s+1}$$

$$G_d(s) = \frac{1}{8s+1} \quad \text{with } D(s) = \frac{1}{s} \tag{9.6}$$

$$G_c(s) = K_c\left[E(s) + \frac{E(s)}{T_I s} - T_d s\,\mathrm{CV}(s)\right]$$

The mathematical optimization must be performed for the two cases. The results of the analysis are given in Table 9.4. The results are similar to the example discussed previously in that the controller gain is decreased, the integral time is increased, and the derivative time is decreased—in this example to zero—as the additional goals are added. The net effect of adding goals 2 and 3 is that total deviation of the controlled variable from its set point (IAE) is larger than that achieved for the nominal process without modelling error. However, the performance indicated by the more comprehensive measure, considering all cases and behavior of both the controlled and manipulated variables, is the best possible

TABLE 9.4

Results for Example 9.1

Case	Controller gain, K_c	Integral time, T_I	Derivative time, T_d	IAE+	
(*a*) Performance, goal 1 alone	3.0	3.7	1.1	1.46	
(*b*) Performance, goals 1–3	1.8	5.2	0.0	2.95	← recommended

+Evaluated for nominal model (without error) without noise.

with a PID control algorithm. Thus, the tuning from case (*b*) is more robust, as will be demonstrated in Example 9.5.

Again we see that there is interaction among the tuning constants. As demonstrated for a simple process in Example 8.5, each tuning constant affects many control performance measures, such as decay ratio and overshoot. Therefore, all tuning constants should be determined simultaneously to obtain the best possible performance within the capability of the PID algorithm.

In conclusion, a very general method has been presented in this section for evaluating controller tuning constants. The method can be applied to any process model and controller algorithm and was applied to the linear, first-order-with-dead-time process and PID controller in this section. The method addresses most control performance issues in a flexible manner, so that the engineer can adapt it to most circumstances by changing a few parameters in the control objective definition, such as the magnitude of the model errors or the allowable variability of the manipulated variable. However, an optimization must be performed for each individual problem, which could be very time-consuming. Thus, the next section describes how controller tuning can be performed quickly in many situations using correlations developed with the optimization procedure.

9.4 □ CORRELATIONS FOR TUNING CONSTANTS

The purpose of tuning correlations is to enable the engineer to calculate tuning constants for many process applications that simultaneously achieve the three goals defined in Section 9.2 without performing the optimization. Correlations for tuning constants will reduce the engineering effort in controller tuning, and, perhaps more importantly, the correlations will show how the controller constants depend on feedback process dynamics. For the correlations developed in this section, the tuning goals will be those defined in Table 9.1 and used in the previous example:

1. Minimize IAE
2. ±25% (correlated) change in the process model parameters
3. Limits on the variation of the manipulated variable

The correlation should provide values for K_c, T_I, and T_d based on values in a process dynamic model. The general approach is to select a model structure and determine the *dimensionless parameters* that define the closed-loop dynamic response. To provide simple, yet general correlations, the process model must have a small number of parameters. Modelling examples in Chapter 6 demonstrated that many processes can be represented by a first-order-with-dead-time transfer function; therefore, this model structure is used in developing the tuning correlations:

$$G_v(s)G'_p(s)G_s(s) \approx G_p(s) = \frac{K_p e^{-\theta s}}{1+\tau s} \tag{9.7}$$

Since the control response is determined by the closed-loop transfer function, the form of the correlation is determined from this transfer function:

$$\frac{CV(s)}{D(s)} = \frac{G_d(s)}{1+G_c(s)G_p(s)} = \frac{G_d(s)}{1+K_c\left(1+\dfrac{1}{T_I s}+T_d s\right)\left(K_p\dfrac{e^{-\theta s}}{1+\tau s}\right)} \tag{9.8}$$

Every process responds with a different "speed," which can be characterized by the time for a step response to achieve 63 percent of its final value. For a first-order-with-dead-time process, this time is $(\theta+\tau)$. Dividing the time by this value "scales" all processes to the same speed, so that one set of general correlations can be developed. The relationships are

$$t' = \frac{t}{\theta+\tau} \qquad s = \frac{s'}{\theta+\tau} \tag{9.9}$$

Substituting the modified Laplace variable for the time-scaled equation gives

$$\frac{CV(s')}{D(s')} = \frac{G_d(s')}{1+K_cK_p\left(1+\dfrac{1}{T_I s'/(\theta+\tau)}+\dfrac{T_d s'}{\theta+\tau}\right)\left(\dfrac{e^{-\theta s'/(\theta+\tau)}}{1+\tau s'/(\theta+\tau)}\right)} \tag{9.10}$$

> The resulting equation has one parameter that characterizes the feedback process dynamics, $\theta/(\theta+\tau)$, which we shall term the *fraction dead time.*

This parameter indicates what fraction of the total time needed for the open-loop process step response to reach 63 percent of its final value is due to the dead time; it has values from 0.0 to 1.0. For example, the base case process data for Figure 9.1 had $\theta = 5$ and $\tau = 5$; thus, the fraction dead time was 0.5. Note that $\tau/(\theta+\tau)$ is not independent, because $\tau/(\theta+\tau) = 1-\theta/(\theta+\tau)$.

Analysis of equation (9.10) also demonstrates that the controller tuning constants and process dynamic model parameters appear in the following *dimensionless* forms:

$$\begin{aligned} \text{Gain} &= K_cK_p \\ \text{Integral time} &= T_I/(\theta+\tau) \\ \text{Derivative time} &= T_d/(\theta+\tau) \end{aligned} \tag{9.11}$$

These relationships are consistent with a common-sense interpretation of the feedback controller relationships. The dimensionless gain involves the magnitude of the change in the manipulated variable to correct for an error and should be related to the process gain. Also, proportional mode has no time dependence. The dimensionless integral time and derivative times involve the time-dependent behavior of the controlled variable and should be related to the dynamics or "time scale" of the process.

The disturbance model is assumed to be the same as the feedback process model; that is, $G_d(s) = G_p(s)$. Noise is assumed to be present in the controlled variable, as discussed in Section 9.3 and defined in Appendix E. The resulting transfer function has only one parameter that is entirely a function of the process [i.e., the fraction dead time $\theta/(\theta + \tau)$]; the tuning constants, expressed in the dimensionless forms in equation (9.11), also influence the dynamic performance. For the control objectives and process model (with error estimate) defined in Table 9.1, the tuning correlations are developed by (1) selecting various values of the fraction dead time in its possible range of 0 to 1 and (2) optimizing the control performance for each value by adjusting the dimensionless tuning constants.

The results for the disturbance response are plotted in Figure 9.5*a* through *c*. The correlations indicate that a high controller gain is appropriate when the process has a small fraction dead time and that the controller gain generally decreases as the fraction dead time increases. This makes sense, because processes with longer dead times are more difficult to control; thus, the controller must be detuned. The dimensionless derivative time is zero for small fraction dead time and increases for longer dead times to compensate for the lower controller gain. The dimensionless integral time remains in a small range as the fraction dead time increases.

The same procedure can be performed for the other major input forcing: set point changes. All of the assumptions and equation simplifications are the same, and the set point is assumed to change in a step. The resulting correlations are presented in Figure 9.5*d* through *f*. The tuning constants have the same general trends as the fraction dead time increases. The selection of whether to use the disturbance or set point correlations depends on the dominant input variation experienced by the control system.

The range of model errors, ±25 percent, is reasonable when all parameters are significantly different from zero. However, when this percentage error is used, a very small dynamic parameter would also have a very small associated error, which may not be realistic. Because an underestimation of the error would generally lead to a controller that is too aggressive, and because the controller for $\theta/(\theta+\tau) = 0.10$ is already quite aggressive, the tuning correlations are not extended lower than 0.10, and the recommended tuning constant values are shown by the lines maintaining the constant values for $\theta/(\theta + \tau)$ from 0.10 to 0. These values can be improved through fine-tuning, if required, as described later in this chapter.

The tuning correlations presented in this section were developed by Ciancone and Marlin (1992) and will be referred to subsequently as the *Ciancone correlations*. The controller tuning method using the Ciancone correlations consists of the following steps:

1. Ensure that the performance goals and assumptions are appropriate.
2. Determine the dynamic model using an empirical method (e.g., the process reaction curve), giving K_p, θ, and τ.
3. Calculate the fraction dead time, $\theta/(\theta + \tau)$.
4. Select the appropriate correlation, disturbance, or set point; use the disturbance if not sure.
5. Determine the dimensionless tuning values from the graphs for $K_c K_p$, $T_I/(\theta + \tau)$, and $T_d/(\theta + \tau)$.
6. Calculate the dimensional controller tuning, e.g., $K_c = (K_c K_p)/K_p$.
7. Implement and fine-tune as required (see Section 9.5).

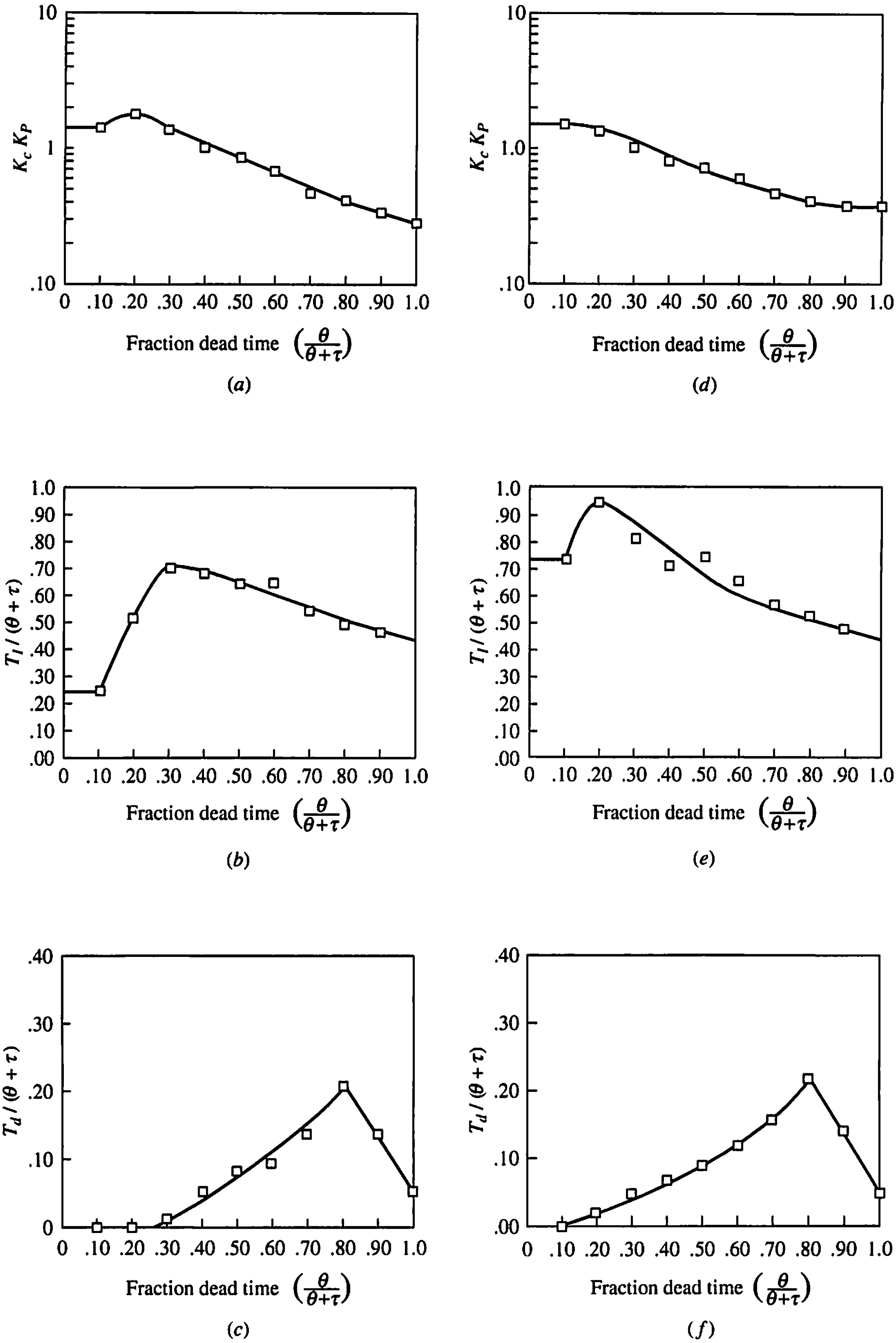
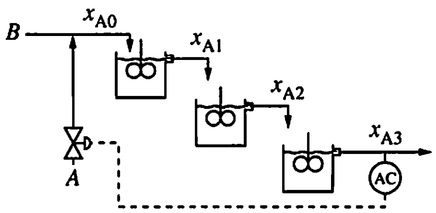


FIGURE 9.5

Ciancone correlations for dimensionless tuning constants, PID algorithm. For disturbance response: (*a*) control system gain, (*b*) integral time, (*c*) derivative time. For set point response: (*d*) gain, (*e*) integral time, (*f*) derivative time.

The reader should recall the likely accuracy in the dynamic model when tuning a PID controller. The gain, time constant, and dead time from empirical identification have significant errors (20 percent is not uncommon); therefore, precise values from the correlations are not required, because small errors in reading the plot are insignificant when compared with the modelling errors. The use of the correlations is demonstrated in the following examples.



EXAMPLE 9.2.

Determine the tuning constants for a feedback PID controller applied to the three-tank mixing process for a disturbance response (step in x_{AB}) using the Ciancone tuning correlations.

The first step is to fit a first-order-with-dead-time model to the process, which was done using the process reaction curve method in Example 6.4. The results were $K_p = 0.039$ %A/% valve opening; $\theta = 5.5$ min; and $\tau = 10.5$ min. Then, the independent parameter is calculated as $\theta/(\theta+\tau) = 0.34$. The dependent variables are determined from Figure 9.5*a* through *c*, and subsequent tuning constants are calculated as follows:

$$K_c K_p = 1.2 \qquad K_c = 1.2/.039 = 30\% \text{ open/\%A}$$
$$T_I/(\theta+\tau) = 0.69 \qquad T_I = 0.69(16) = 11 \text{ min}$$
$$T_d/(\theta+\tau) = 0.05 \qquad T_d = 0.05(16) = 0.8 \text{ min}$$

The dynamic response of the feedback system to a step feed composition disturbance of magnitude 0.80%A occurring at time = 20 is given in Figure 9.6, which results in an IAE of 7.4. The dynamic response is "well behaved"; that is, the

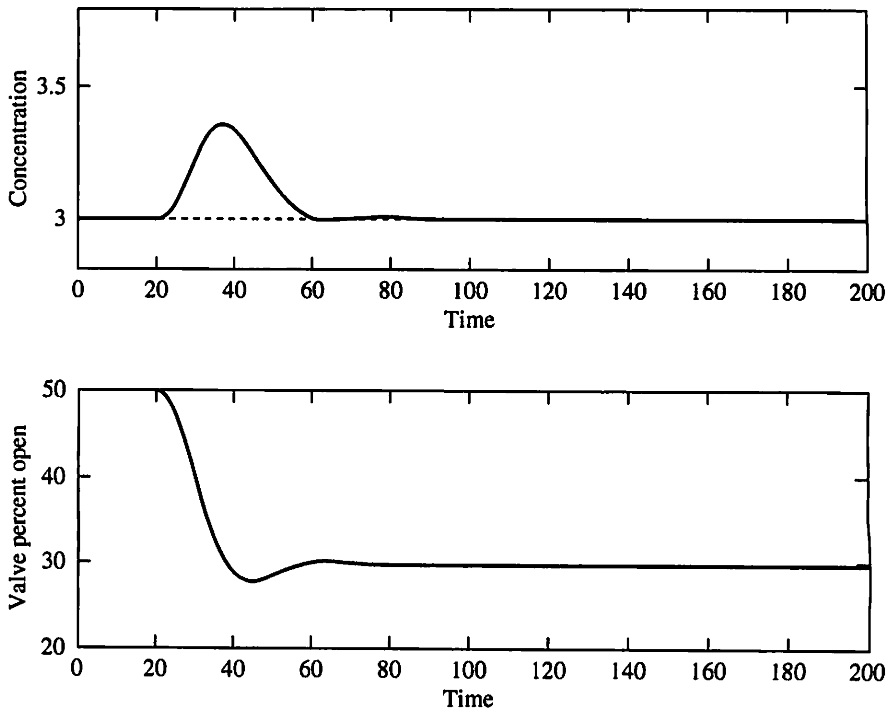


FIGURE 9.6

Dynamic response of three-tank process and PID controller with tuning from Example 9.2.

controlled variable returns to its set point reasonably quickly without excessive oscillations, and the manipulated variable does not experience excessive variation.

The result in Example 9.2 shows that the correlations, which were developed for first-order-with-dead-time plants, provide reasonable tuning for plants with other structures as long as the feedback process dynamics can be approximated well with a first-order-with-dead-time model. Recall that overdamped processes with monotonic S-shaped step responses are well represented by first-order-with-dead-time models.

EXAMPLE 9.3.

When developing the correlations, the assumption was made that the disturbance transfer function was the same as the process feedback transfer function. Evaluate the tuning correlations for the same three-tank system considered in Example 9.2 with a different disturbance time constant.

Original disturbance transfer function:

$$G_d(s) = \frac{1}{(5s+1)^3}$$

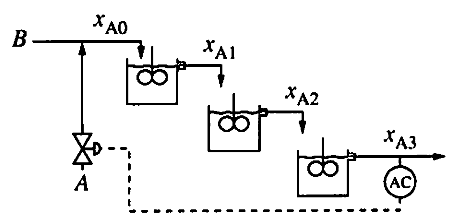
Altered disturbance transfer function:

$$G_d(s) = \frac{1}{(5s+1)}$$

The altered transfer function would occur if the disturbance entered in the last tank of the three. The resulting transient of the system under closed-loop control is plotted in Figure 9.7. As would be expected, the response is different, with the faster disturbance resulting in poorer control with respect to the maximum deviation and IAE, which increased to 8.3. The slightly poorer control performance is the result of a more difficult process, due to the faster disturbance, being controlled. Note that the correlation tuning constants give reasonably good, although not "optimal," performance even when the disturbance transfer function differs significantly from the feedback transfer function.

EXAMPLE 9.4.

The correlations have been developed assuming that the process is linear, and it has accounted for changes in the process dynamics through the range of model error considered. In this example a process is considered in which the nonlinearities influence the dynamics during the transient response. The three-tank mixer described in Example 7.2 is nonlinear if the flow of stream B changes, as seen by the fact that the time constants and gain in the linearized model depend on F_B. Determine the tuning and dynamic response for the situation in which F_B changes from its base value of 6.9 m³/min to 5.2 m³/min and returns to its base value.



The tuning for the initial condition has been determined in Example 9.2. Before evaluating the dynamic response, it is worthwhile determining the change in the process dynamics resulting from the change in F_B, which is summarized here for the models linearized about the base and disturbed steady states:

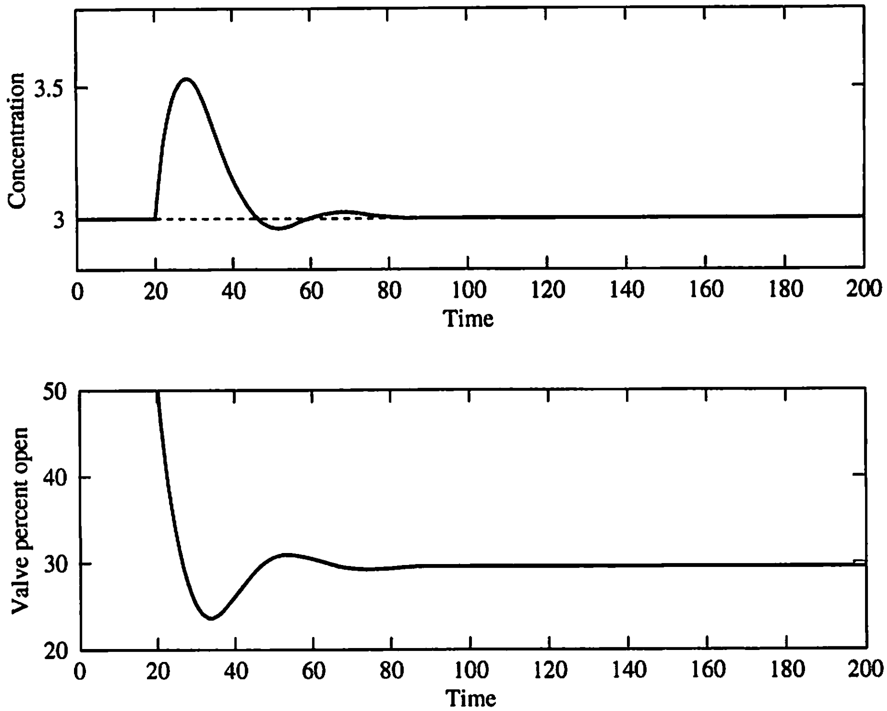


FIGURE 9.7

Dynamic response of three-tank mixing process with faster disturbance dynamics from Example 9.3.

Parameter	Dependence on process	Base case value ($F_B = 6.9$)	Disturbed case value ($F_B = 5.2$)
Time constant, τ (min)	$V/(F_B + F_A)$	5.0	6.6
Steady-state gain, K_P (%A/% open)	$K_v[(x_A)_A - (x_A)_B]F_B/(F_B + F_A)^2$	0.039	0.051

The process model changes during the transient, and it would be proper to correct the tuning. However, it is not possible to change the tuning for all disturbances, many of which are not measured; thus, the base case tuning is used during the entire transient in this example. The results are plotted in Figure 9.8. Note that the first transient in response to a decrease in flow experiences rather oscillatory behavior; this is because the process dynamics are slower because of the change in operations, and consequently the tuning is too aggressive. When returning to the base case, the tuning is only slightly underdamped, because the conditions are close to the dynamics for which the tuning constants were determined. Even for this significant change in process dynamics, the PID algorithm with tuning from the Ciancone correlations provides acceptable performance. Thus, the system is robust to disturbances of the magnitude considered in this example. However, larger changes in process operation would result in larger model variation and could seriously degrade performance or even cause instability. One method for maintaining good control performance when large changes in dynamics occur is to continually recalculate the tuning constant values based on measured disturbances. This method is explained in Section 16.3.

The results of the tuning studies lead to two important observations concerning the effects of process dynamics on tuning. First, the controller should be detuned; that is, the feedback adjustments should be reduced as the fraction dead time of the feedback process increases. Thus, we conclude that dead time in the feedback loop results in reduced or slower feedback adjustments and, presumably, poorer control. Theoretical justification for this result is presented in Chapter 10, and the effect on feedback performance is confirmed in Chapter 13.

The second observation is that two models, the feedback process $G_p(s)$ and the disturbance process $G_d(s)$, both affect the tuning; this is determined by comparing the results for a process disturbance, which enters through a first-order time constant, with those for a set point change, which is a perfect step. However, the major influence on tuning is normally from the *feedback dynamics*, and again, theoretical justification for this result will be presented in the next chapter. Other studies by Hill et al. (1987) showed that the tuning is insensitive to the disturbance time constant when $\tau_d > \tau$; thus, the differences between Figure 9.5*a* through *c* and 9.5*d* through *f* typically represent the maximum change in tuning in response to different disturbance types.

In many control applications the derivative mode is not employed. This is the case if the measurement signal has considerable noise. Also, the tuning correlations demonstrate that the derivative time is very small when the fraction dead time is small. Thus, tuning correlations for a proportional-integral (PI) controller are provided in Figure 9.9*a* and *b* for a disturbance and set point responses. Note that it would not be correct to use the PID values and simply set the derivative time T_d to zero, because of the interaction between the tuning constant values, although the correlations in Figure 9.9 are close to those in Figure 9.5 because of the small values of the derivative time in Figure 9.5.

The tuning correlations presented in Figures 9.5 and 9.9 depend on the goals specified for the control performance. It is interesting to compare the results to a different set of goals. One of the earlier studies using an optimization procedure was performed by Lopez et al. (1969). In their study the goal was simply to minimize the IAE (our goal 1), without concern for potential variation in feedback dynamics or limitations on manipulated-variable transient behavior. Their results are presented in Figure 9.10*a* and *b* and are applied in the following example.

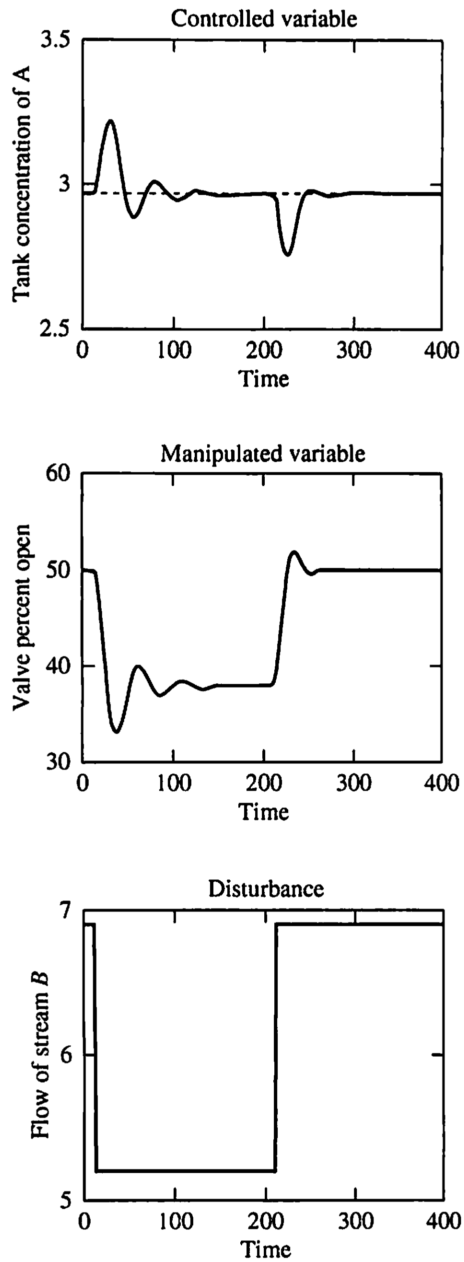


FIGURE 9.8

Dynamic response for Example 9.4 in which the feedback dynamics change due to the disturbance.

EXAMPLE 9.5.

The altered mixing process in Figure 9.1, with the transfer function given below, is to be controlled with a PI controller. Calculate the tuning constants according to correlations in Figure 9.9*a* and *b* and 9.10 using the nominal model given below. Calculate the transient responses to a step disturbance of 2%A in feed composition at time = 7 for (*a*) the nominal feedback process and (*b*) an altered plant as defined below. Note that the nominal and actual plants have the same steady-state gain and "speed of response," as measured by the time to reach 63 percent of their steady-state value to a step change input; they differ only in their fraction dead time.

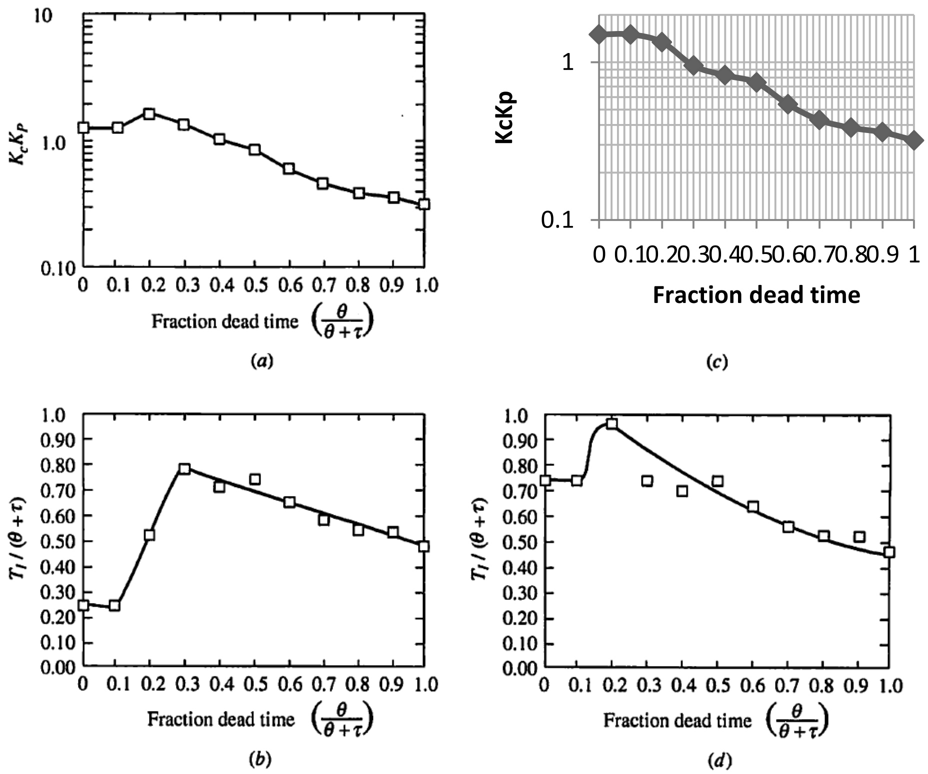
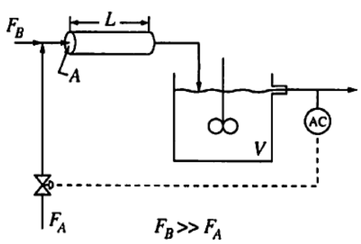


FIGURE 9.9

Ciancone correlations for dimensionless tuning constants, PI algorithm. For disturbance response: (*a*) controller gain and (*b*) controller integral time. For set point response: (*c*) controller gain and (*d*) controller integral time.



Nominal plant:

$$G_p(s) = \frac{2.0e^{-2s}}{8s+1}$$
$$G_d(s) = \frac{1.0}{8s+1}$$
$$\theta + \tau = 10$$
$$\frac{\theta}{\theta+\tau} = 0.2$$

Altered plant:

$$G_p(s) = \frac{2.0e^{-3s}}{7s+1}$$
$$G_d(s) = \frac{1.0}{7s+1}$$
$$\theta + \tau = 10$$
$$\frac{\theta}{\theta+\tau} = 0.3$$

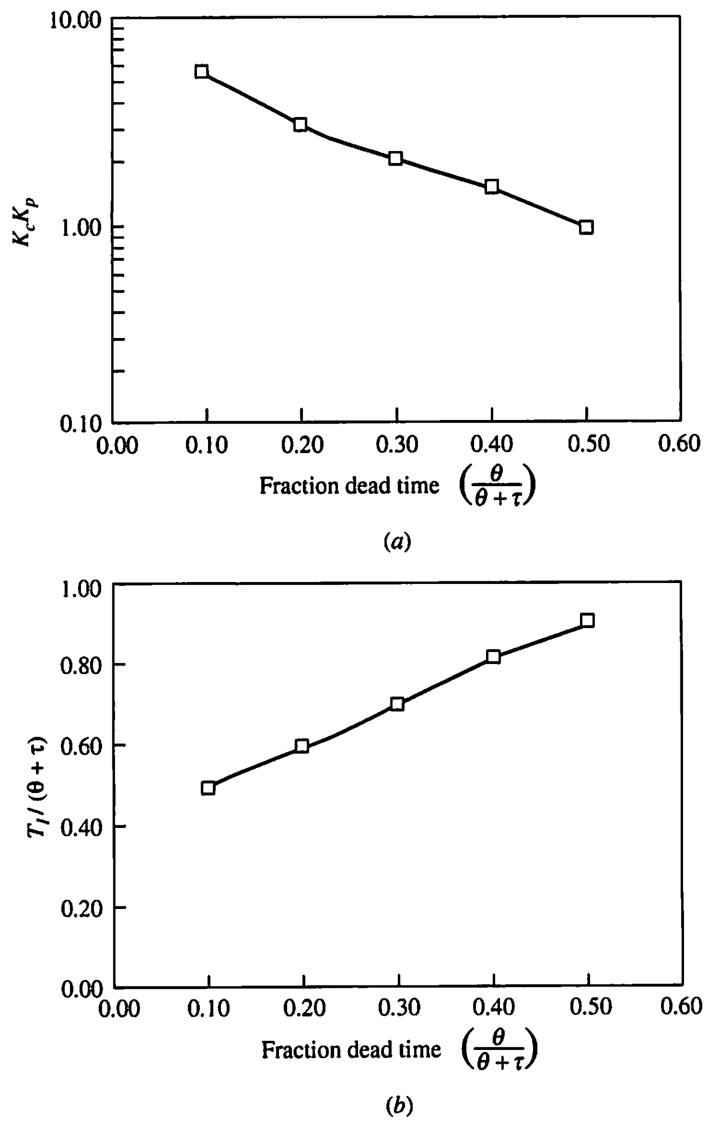


FIGURE 9.10

Lopez et al. (1969) tuning correlations for minimizing the IAE for a PI controller in response to a disturbance.

The tuning constant values can be calculated for each correlation from the charts using the nominal model as

	Ciancone	Lopez	
K_c	0.9	1.5	%open/%A
T_I	5.2	6.0	min

The closed-loop dynamic responses are given in Figure 9.11*a* through *d*, and the control performance measure of IAE is summarized as

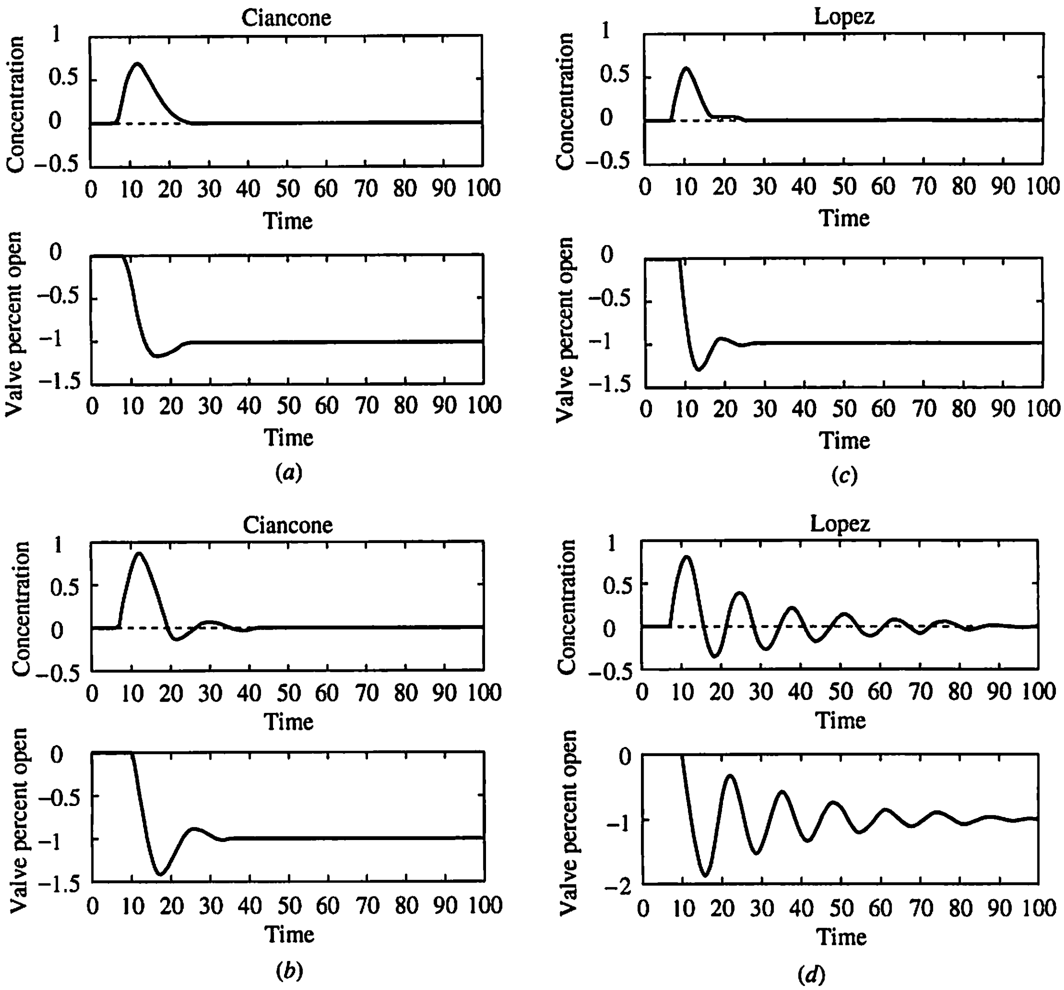


FIGURE 9.11

Dynamic responses of deviation variables. With Ciancone tuning: (*a*) nominal plant, (*b*) altered plant. With Lopez tuning: (*c*) nominal plant, (*d*) altered plant.

	Ciancone	Lopez
IAE for nominal plant	5.9	4.0
IAE for altered plant	7.6	14.5 ← Ciancone gives robustness to model errors

These results should be anticipated from the control objectives used to derive the correlations. The Lopez correlation minimized IAE without consideration for model error. Thus, it performs best when the plant model is known perfectly, but it is unacceptably oscillatory and tends toward instability for even the modest model error considered in this example. The Ciancone correlations determined the tuning to perform well over a range of process dynamics; thus, the performance does not degrade as rapidly with model error.

The results of this section show that simple PID tuning correlations can be developed for processes that can be approximated by a first-order-with-dead-time model. Selection of the proper correlation depends on the control performance goals. If the situation indicates that very accurate knowledge of the process is available and there is no concern for the manipulated-variable variation, the best performance (i.e., lowest IAE of the controlled variable with PI feedback) is obtained using the Lopez correlations; however, the control system with these tuning constants will not perform well if the process model has significant error or if the measurement has significant noise. As the control performance goals are defined more realistically for typical plant situations, the resulting tuning allows for more modelling error and for some limitation on the manipulated-variable variation, and the resulting correlations have a broader range of good performance. This is an important factor for control systems that function continuously for months or years as plant conditions change. Thus, the Ciancone correlations are recommended here as a starting point for most control systems.

> Tuning correlations have been developed as a function of fraction dead time for a PID controller, a first-order-with-dead-time process, and typical control objectives. These are recommended for obtaining initial tuning constant values when the plant situation matches the factors in Table 9.1.

It is important to recognize that no claim is made for optimality in the real world, although an optimization method was used to determine the solution to the mathematical problem. The Ciancone correlations simply used a realistic definition of control performance to determine tuning. Also, while examples have shown that the correlations are valid for different disturbance model parameters and model errors, extrapolation beyond the defined conditions of the correlation (Table 9.1) must be done with care.

9.5 ◻ FINE-TUNING THE CONTROLLER TUNING CONSTANTS

The tuning constants calculated according to any method—optimization, correlations, or the stability analysis in the next chapter—should be considered to be *initial* values. These values can be applied to the process to obtain empirical information on closed-loop performance and modified until acceptable control performance is obtained. Determining modifications based on initial dynamic responses, often termed *fine-tuning,* is necessary because of errors in the base case process model and simplifications in the tuning method. A fine-tuning method is described here for a process being controlled by a PI control algorithm. This method is easy to perform and gives additional insight into the way the controller modes combine when controlling a process.

After the initial tuning constants have been calculated and entered into the algorithm, the controller's status switch can be placed in the automatic position to allow the controller to perform its calculation and adjust the final element. Then, the response to a set point change is diagnosed to determine whether the tuning is satisfactory. A *set point change* is considered here because

1. It can be introduced when the diagnosis is performed.
2. A simple time-dependent input disturbance—a step—is easy to achieve.
3. The magnitude can be selected by the engineer.
4. The effects of the proportional and integral mode calculations can be separated, which greatly simplifies the diagnosis of the controller behavior.

The step response of a control system with a well-tuned PI controller is given in Figure 9.12. The first important feature is the immediate change in the manipulated variable when the set point is changed. This is due to the proportional mode and is equal to $K_c \Delta E(t)$, which is equal to $K_c \Delta \mathrm{SP}(t)$. This initial change is typically 50 to 150 percent of the change at the final steady state. The second feature is the delay, due to the dead time, between when the set point is changed and when the controlled variable initially responds. No controller can reduce this delay to be less than the dead time. During the delay the error is constant, so that the proportional term does not change, and the magnitude of the integral term increases linearly in proportion to $K_c E(t)/T_I$. When the controlled variable begins to respond, the proportional term decreases, while the integral term continues to increase. At the end of the transient response the proportional term, being proportional to error, is zero, and the integral term has adjusted the manipulated variable to a value that reduces offset to zero.

The value of this interpretation can be seen when an improperly tuned controller, giving the response in Figure 9.13, is considered. The control response seems slow, resulting in a large IAE and a long time to return to the set point. Analysis of the transient indicates that the initial change in the manipulated variable when the set point is changed, termed the proportional "kick," is only about 30 percent of the final value, which indicates too small a value for the controller gain. The conclusion for the diagnosis is that the control system performance can be improved by increasing the controller gain, most likely in several moderate steps, with a plant test at each step to monitor the results of the changes. The

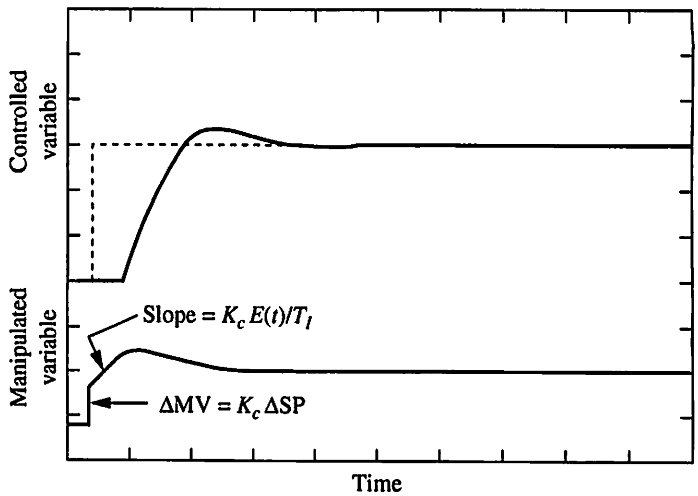


FIGURE 9.12

Typical set point response of a well-tuned PI control system.

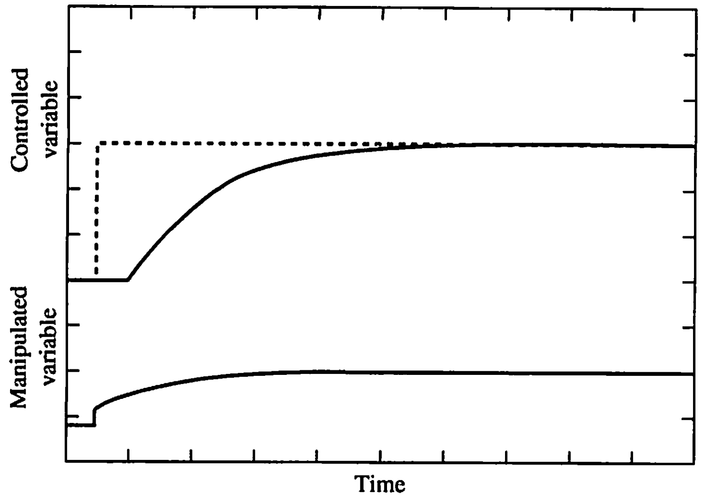


FIGURE 9.13

Example of a dynamic response of a PI control system with the controller gain too small.

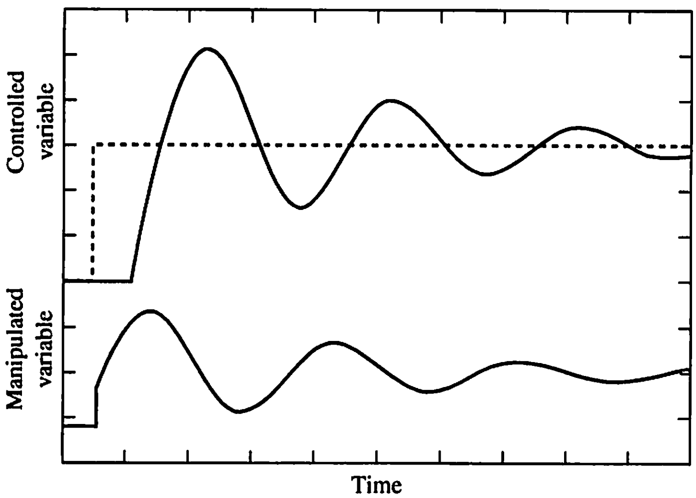


FIGURE 9.14

Dynamic response of the control system in Example 9.6.

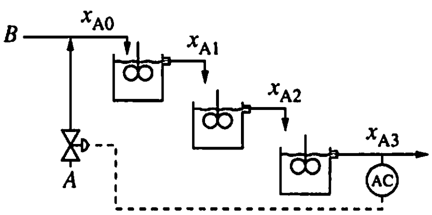
substantially improved performance of the control system with the controller gain increased by a factor of 2.5 is shown in Figure 9.12.

EXAMPLE 9.6.

A PI controller was not providing acceptable control performance. Preliminary analysis indicated that the sensor and control valve were functioning properly, so a step change was introduced to its set point. The response is given in Figure 9.14. Diagnose the performance, and suggest corrective action.

Solution. The transient response is highly oscillatory, indicating a controller that is too aggressive. The cause could be too large a controller gain, too short an integral time, or both. The immediate proportional change is only about 70 percent of the final change in the manipulated variable; therefore, the controller gain is in a reasonable range, is certainly not too large, and should not cause oscillatory behavior. The conclusion is that the integral time is too short. The transient response with double the integral time is that shown in Figure 9.12, confirming that reasonably good control performance can be achieved by changing only the integral time.

EXAMPLE 9.7.



The three-tank mixing control system has been tuned initially, and the system's dynamic response to a set point change is given in Figure 9.15*a*. Note that the measured concentration experiences many small disturbances because of changing inlet concentrations and flows in the process as well as measurement error. This noisy data more closely represents empirical data from process plants than do the ideal simulations in Figures 9.12 through 9.14. The control objectives have two unique aspects in this example, which are different from the general objectives considered so far but are not unusual in the process industries.

1. The downstream process is sensitive to oscillations in the concentration. Therefore, the controlled concentration should not experience overshoot.
2. The plant that supplies component A functions better with a smooth operation. Therefore, high-frequency variation in the manipulated variable is to be minimized.

The initial tuning constants are $K_c = 45\%$ opening/%A, $T_I = 11.0$ minutes, and $T_D = 0.8$ minute. Suggest changes to the tuning constant values that will improve the performance.

Solution. The large, high-frequency variation in the manipulated variable is caused to a large extent by the noisy measurement and the derivative mode. Therefore, the first suggestion would be to reduce the derivative time to zero. Next, the controlled variable overshoots its set point, which can be prevented by making the controller feedback action less aggressive. Reducing the controller gain will slow the response and also slightly reduce the high-frequency variation of the manipulated variable, both desirable effects. The resulting tuning constants, which could be arrived at after several trials, are $K_c = 15$, $T_I = 11$, and $T_d = 0$. A much more satisfactory dynamic response—that is, one that more closely satisfies the stated objectives for this example—was obtained with these tuning constants, as shown in Figure 9.15*b*. Note that the much smoother performance was achieved with only a small increase in IAE, which changed from 11.6 to 12.9.

These fine-tuning examples demonstrate that

> **Analysis of the responses of the controlled and *manipulated* variables to a step change in the set point provides valuable diagnostic information on the causes of good and poor control performance, allowing the performance to be tailored to unique control objectives.**

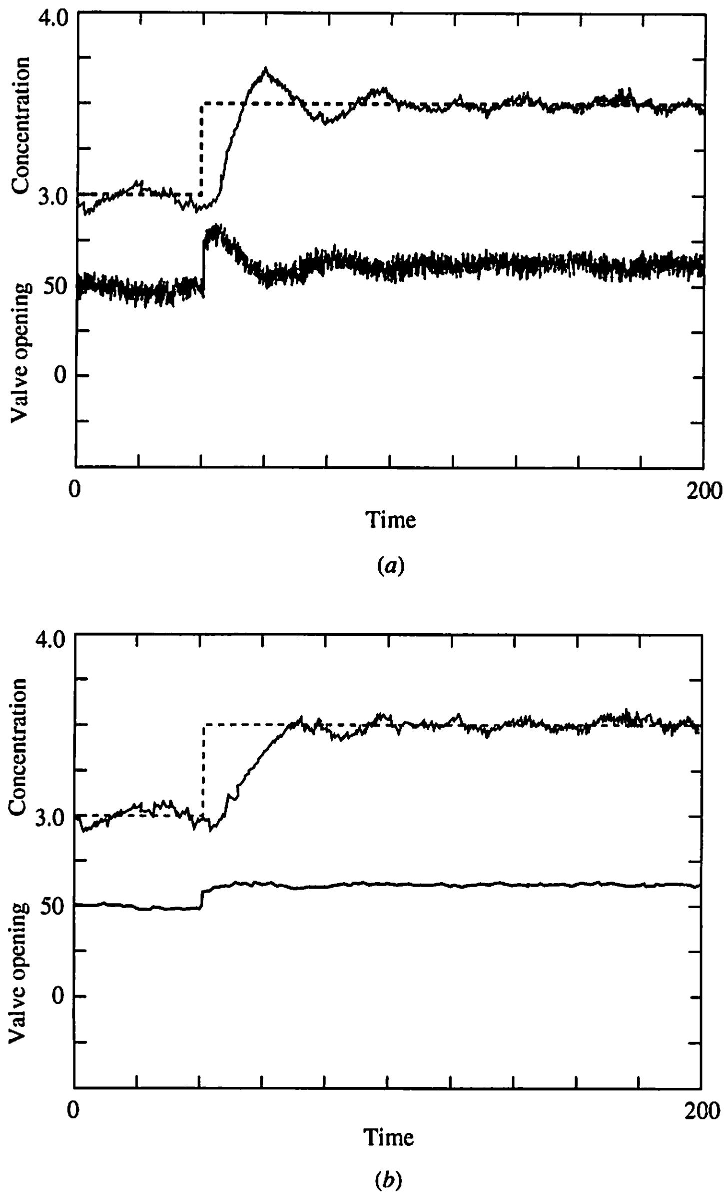


FIGURE 9.15

Dynamic responses of feedback control system in Example 9.7: (*a*) initial (IAE = 11.6); (*b*) after fine-tuning (IAE = 12.9).

Again, we see that both the controlled and manipulated variables must be observed when analyzing the performance of feedback control systems; complete diagnosis is not possible without information on both variables.

9.6 ▫ CONCLUSIONS

The starting point for feedback control consists of the control objectives, here specified as three goals. These goals encompass the major factors in process control performance; the specific parameters used (e.g., percent model error and limits on manipulated-variable variation) can be selected to match a specific problem.

Control performance must be defined with respect to all important plant operating goals. In particular, desired behavior of the controlled and manipulated variables must be defined for expected disturbances, model errors, and noisy measurements.

A simple variable reduction of the closed-loop transfer function, based on dimensional analysis, can be employed in extending the optimization to general tuning correlations. These correlations are applicable only to those systems for which the underlying assumptions are valid: The process should be well represented by a first-order-with-dead-time model, the model errors should be in the assumed range, and the desired controlled and manipulated behavior should be similar to the objectives stated in Table 9.1. Examples have demonstrated that the process does not have to be perfectly first-order with dead time to achieve acceptable dynamic responses using the tuning correlations.

A three-step tuning procedure would combine methods in previous chapters with methods in this chapter. The first step would be to determine the feedback process model $G'_p(s)G_v(s)G_s(s)$ by fundamental modelling or empirical modelling, using either the process reaction curve or a statistical identification method. Industrial controls are most often based on empirical models. In the second step, the initial tuning constant values would be determined; typically the values would be determined from the general correlations, but an optimization calculation could be performed for processes that are not adequately modelled by a first-order-with-dead-time model. The third step involves a test of the closed-loop control system and fine-tuning, if necessary. The set point step change provides separate information on the proportional and integral modes to facilitate diagnosis and corrective action.

The dynamic behavior of both the controlled and the manipulated variables is required for evaluating the performance of a feedback control system.

The reader should clearly recognize the meaning of the term *optimum*. It is used here to mean results (i.e., tuning constant values) that are determined so that certain mathematical criteria are satisfied. The criteria are goals 1 to 3. Naturally, the relationships in Table 9.1 were selected to represent the true control situation closely for the majority of cases. However, control performance has many facets, from safety through profit; therefore, it is sometimes difficult to condense all of the critical factors into one measure of control performance. Even if the mathematical objectives successfully represent the true desired performance, the results will be satisfactory only when the parameters in the mathematical formulation specify the desired behavior. These parameters, such as the controlled-variable measurement noise, the expected plant model error, and the allowable manipulated-variable variation, are never known exactly. Therefore, although the mathematical solution is "optimum," the usefulness of the results depends on the accuracy of the input data.

Practically, the values from the optimization or correlations are used as *initial values* to be applied to the physical system and improved based on empirical performance during fine tuning.

Remember, when tuning a feedback controller, where you start is not as important as where you finish!

Finally, the three tuning constants in the PID algorithm all influence the dynamic behavior of the closed-loop system. They must be determined simultaneously, because of this interaction.

It should be apparent that the tuning approach using optimization is not limited to PID controllers; if another algorithm were suggested, its parameters could be optimized by the same procedure. In fact, some results for other feedback controllers are presented in Chapter 19.

The techniques in this chapter provide practical methods for controller tuning that are applicable to many processes. However, they do not provide important explanations to key questions such as

1. Why do the tuning correlations have the shapes in Figure 9.5?
2. Why can a control system become unstable, and how can we predict when this will occur?
3. How does the controller change the dynamic behavior of an open-loop system to that of a closed-loop system?

Methods for answering these more fundamental questions are addressed in the next chapter.

REFERENCES

Ciancone, R., and T. Marlin, "Tune Controllers to Meet Plant Objectives," *Control, 5,* 50–57 (1992).

Edgar, T., and D. Himmelblau, *Optimization of Chemical Processes,* McGraw-Hill, 1988.

Fertik, H., "Tuning Controllers for Noisy Processes," *ISA Trans., 14, 4,* 292–304 (1975).

Hill, A., S. Kosinari, and B. Venkateshwa, "Effect of Disturbance Dynamics on Optimal Tuning," *Instrumentation in the Chemical and Petroleum Industries,* Vol. 19, Instrument Society of America, Research Triangle Park, NC, 89–97 (1987).

Lopez, A., P. Murrill, and C. Smith, "Tuning PI and PID Digital Controllers," *Instr. and Contr. Systems, 42,* 89–95 (Feb. 1969).

Zumwalt, R., *EXXON Process Control Professors' Workshop,* Florham Park, NJ, 1981.

ADDITIONAL RESOURCES

Other common forms of the PID control algorithm and conversions of tuning constants for these forms are given in

Witt, S., and R. Waggoner, "Tuning Parameters for Non-PID Three Mode Controllers," *Hydro. Proc., 69,* 74–78 (June 1990).

Analytical solutions for optimal tuning constant values for PID controllers can be obtained for some continuous control systems, specifically those involving processes without dead time. They can also be obtained for digital controllers for processes with dead time. References for analytical methods are given below; however, since such solutions are possible only with intensive analytical effort for limited control performance specifications, numerical methods are used in this chapter.

Jury, E., *Sample-Data Control Systems* (2nd ed.), Krieger, 1979.
Newton, G., L. Gould, and J. Kaiser, *Analytical Design of Linear Feedback Controls,* Wiley, New York, 1957.
Stephanopoulos, G., "Optimization of Closed-Loop Responses," in Edgar, T. (ed.), *AIChE Modular Instruction Series, Vol. 2, Module A2.5,* 26–38 (1981).

Background on mathematical principles and numerical methods of optimization can be obtained from many reference books, for example:

Reklaitis, G., A. Ravindran, and K. Ragsdell, *Engineering Optimization, Methods and Applications,* Wiley, New York, 1983.

Many other studies have been performed on optimizing time-domain control system performance, for example:

Bortolotto, G., A. Desages, and J. Romagnoli, "Automatic Tuning of PID Controllers through Response Optimization over Finite-Time Horizon," *Chem. Engr. Comm., 86,* 17–29 (1989).
Gerry, J., "Tuning Process Controllers Starts in Manual," *InTech,* 125–126 (May 1999).

The diagnostic fine-tuning method described in this chapter is limited to step changes in the controller set point. A powerful method for diagnosing feedback controller performance is based on statistical properties of the controlled and manipulated variables. The method, which establishes the approach to best possible control and identifies reasons for poor performance, is given in

Desborough, L., and T. Harris, "Performance Assessment for Univariate Feedback Control," *Can. J. Chem. Engr., 70,* 1186–1197 (1992).
Harris, T., "Assessment of Control Loop Performance," *Can. J. Chem. Engr., 67,* 856–861 (1989).
Stanfelj, N., T. Marlin, and J. MacGregor, "Monitoring and Diagnosing Control System Performance—SISO Case," *IEC Res., 32,* 301–314 (1993).

An alternative method of fine-tuning is based on shapes or patterns of response to disturbances. Good and poor responses are identified, and tuning constants are altered accordingly. This method has been applied in an automatic tuning system. For an introduction, see

Kraus, T., and T. Myron, "Self-Tuning PID Controller Uses Pattern Recognition Approach," *Control Eng., 31,* 106–111 (June 1984).

The derivative mode can substantially improve the performance of control loops involving processes that are underdamped or unstable without control. For underdamped systems, see question 8.17. For open-loop unstable processes, see

Cheung, T., and W. Luyben, "PD Control Improves Reactor Stability," *Hydro. Proc., 58,* 215–218 (September 1979).

These questions reinforce the key aspects of dynamic behavior that are considered in defining control performance and how the performance goals and process dynamics influence the controller tuning.

QUESTIONS

9.1. Given the results of the process reaction curve in Figure Q9.1, calculate the PI and PID tuning constants. The process was initially at steady state, and the manipulated variable was changed in a step at time = 0 by +7%.

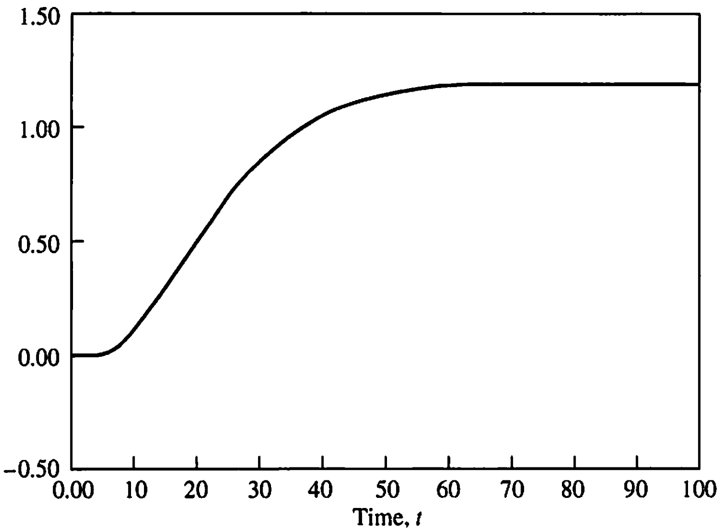


FIGURE Q9.1

9.2. Suppose that control goals different from those in Table 9.1 are specified for the tuning correlations. Predict the effect on the tuning constant values—that is, whether each would increase or decrease from the correlation values from Figure 9.5—for each set of goals.

(a) The only goal is to minimize the IAE for the base case model.
(b) The goals are to minimize IAE for ±25% change in model parameters, without concern for the manipulated-variable variation.
(c) The goals are to minimize IAE for ±50% change in model parameters, with concern for the manipulated-variable variation—unchanged from Table 9.1.

9.3. Confirm the correlation between the linearized model parameters and the process operating conditions in Table 9.3. Calculate the change in flow rate for the specified range of model parameters.

9.4. The dynamic responses shown in Figure Q9.4 were obtained by introducing a step set point change to a PID controller. The dead time of the process is only a few minutes. For each case, determine whether the control is as good as possible and if not, what corrective steps should be taken. Note that the diagnosis of this data would require an exact specification of the control objectives. Use the general objectives considered in Table 9.1 and be as specific as possible regarding the change to the tuning constants.

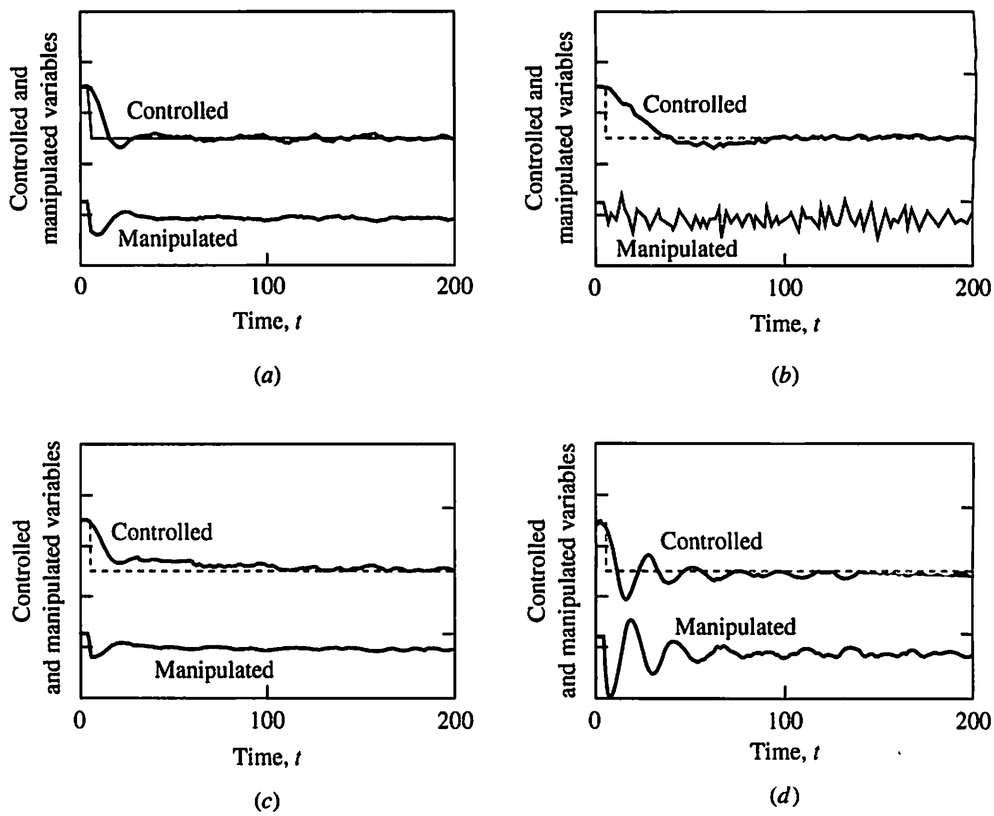


FIGURE Q9.4

9.5. The tuning constants for the three-tank control system are given in Example 9.2. Predict how the optimum tuning constants will change as the following changes are made to the control system. The analysis should be based on principles of process dynamics, tuning factors, and tuning correlations. Be as specific as possible without resolving the optimization problem for each case.

(*a*) A different control valve is installed whose maximum flow is 2.5 times greater than the original valve.
(*b*) The volume of each tank is reduced by a factor of 2.
(*c*) The temperature of stream *B* is increased by 20°C.
(*d*) The set point of the controller is increased to 3.5 percent of component A in the third-tank effluent.
(*e*) Substantial high-frequency noise is present in the measurement of the controlled variable.

9.6. Given the following process reaction curves, for which of the processes is it appropriate to use the general tuning charts in Figure 9.4*a* through *f*? Explain your answer for each case.

(*a*) Figure 3.7 (tank 2 concentration)
(*b*) Figure 3.18
(*c*) Figure 5.5
(*d*) Figure I.5 (Appendix I)
(*e*) Figure 7.3*a*, 7.3*b*
(*f*) Figure 8.4*a*
(*g*) Figure 5.17

9.7. Explain in your own words why the dimensionless parameters are

(*a*) $K_c K_p$.
(*b*) $T_I/(\theta + \tau)$.
(*c*) $T_d/(\theta + \tau)$.

9.8. Derive the closed-loop transfer function for the three-tank mixing process using the analytical (third-order) linearized model in response to a change in the composition in the *A* stream from Example 7.2. Perform a dimensional analysis using the method demonstrated in Section 9.4, determine the key dimensionless parameters, and explain the form of tuning correlations for this model structure and how you would develop them.

9.9. For one or more of the following processes, calculate the PI controller tuning constants by two correlations: Ciancone and Lopez. Compare the expected control performance for both correlations in response to a step change in the controller set point. Under which circumstances would each correlation give the best constants?

(*a*) Question 6.1
(*b*) Question 6.2
(*c*) CSTR in Section 3.6
(*d*) Example 5.1
(*e*) Example I.2 (Appendix I)
(*f*) Example 6.4

9.10. The two series CSTRs in Example 3.3 with the reaction A → products

$$-r_A = 6.923 \times 10^5 e^{-5000/T} C_A$$

with T in K, has its outlet concentration of A, C_{A2}, controlled by adjusting the inlet concentration C_{A0}. The temperature varies slowly between 290 and 315 K. Would this temperature variation require a significant adjustment in controller tuning? Justify your answer with quantitative analysis.

9.11. The three cases used in the tuning optimization are selected to span the range of expected plant operation (i.e., the range of plant model parameters). Suppose that the control engineer knew what percentage of the time that the plant will operate at various operating conditions in the range. Suggest a modification to the optimization method, specifically the objective function, that would include the information on time at each operation in determining the optimum tuning constants.

9.12. The tuning optimization method integrates the equations over a finite time to evaluate the IAE.

(*a*) Write the equations that could be used to evaluate the IAE from the simulation results.

(*b*) Write the equations for the ISE and ITAE that could be used with simulation results. For the ITAE, carefully define when the integration begins (i.e., where time equals zero).

(*c*) Examples in this chapter demonstrated that a poor choice of tuning constant values could lead to an unstable system, with the controlled variable diverging from the solution. What is the theoretical value of the IAE for an unstable control system? How would the optimization system described in this chapter respond if an intermediate set of tuning constants led to an unstable response?

(*d*) Determine the theoretical minimum IAE for controlling an ideal first-order process with dead time in response to a step disturbance.

(*e*) If an analytical expression were available for $CV(t)$, it could be used in tuning. Determine the closed-loop transfer function for a PI controller and a first-order-with-dead-time process, $G_p(s) = K_p e^{-\theta s}/(\tau s + 1)$. For a step set point change, $SP(s) = \Delta SP/s$, solve for $CV(s)$ and invert the Laplace transform to obtain $CV(t)$, if possible.

9.13. Control performance goals are defined in Table 9.1. Propose at least one alternative measure for every entry in the column labeled "Used in This Chapter." Each should involve a different performance measure and not be simply a different numerical value. Discuss the advantages of each entry, the original, and your proposed alternate.

9.14. Tuning constants for a PI controller for the following process are to be determined.

$$G'_p(s)G_v(s)G_s(s) = \frac{7.5e^{-2.3s}}{8.5s + 1} \qquad G_d(s) = \frac{100}{5s + 1}$$

The control objectives are essentially the same as used in this chapter. A colleague has calculated several sets of values for the controller gain and integral time. Determine which of these sets of constants, if any, is acceptable and explain why or why not.

Tuning	Case A	Case B	Case C	Case D
K_c	12	12	0.3	0.3
T_I	6	1	6	1

9.15. Rules for interpreting the control performance are presented in the section on fine-tuning and summarized in Figure 9.12.

(*a*) Discuss the advantages of using a set point change response rather than the disturbance response.

(*b*) Prove the relationships given in Figure 9.12.

(*c*) Demonstrate why the initial change in the manipulated variable is about 50 to 150 percent of its final value. Does this tuning guideline depend on the tuning goals and correlations used?

9.16. Figure 9.2 gives the controlled variable behavior for various values of the controller gain. Sketch the behavior of the manipulated variable you would expect for each case and explain your answers. Also, sketch the variable given here as a function of the controller gain K_c, and explain your answer.

$$\int_0^{\infty} \left(\frac{d(\mathrm{MV})}{dt} \right)^2 dt$$