

# Dynamic Behavior of Typical Process Systems

CHAPTER

5

## 5.1 ■ INTRODUCTION

Examples in the previous two chapters have demonstrated that physical systems, which involve very different physical principles, can have similar dynamic behavior. The concept that a single model type can apply to a wide range of entities, process plants, biological units, economic communities, and so forth provides the basis for “systems” analysis. Thus, it is possible to acquire understanding of a large number of systems from a thorough study of a much smaller number of basic models. In this chapter we study some fundamental model structures that occur frequently in process plants, along with their effects on dynamic behavior. This experience will enable us to recognize the effects of process designs on dynamic behavior.

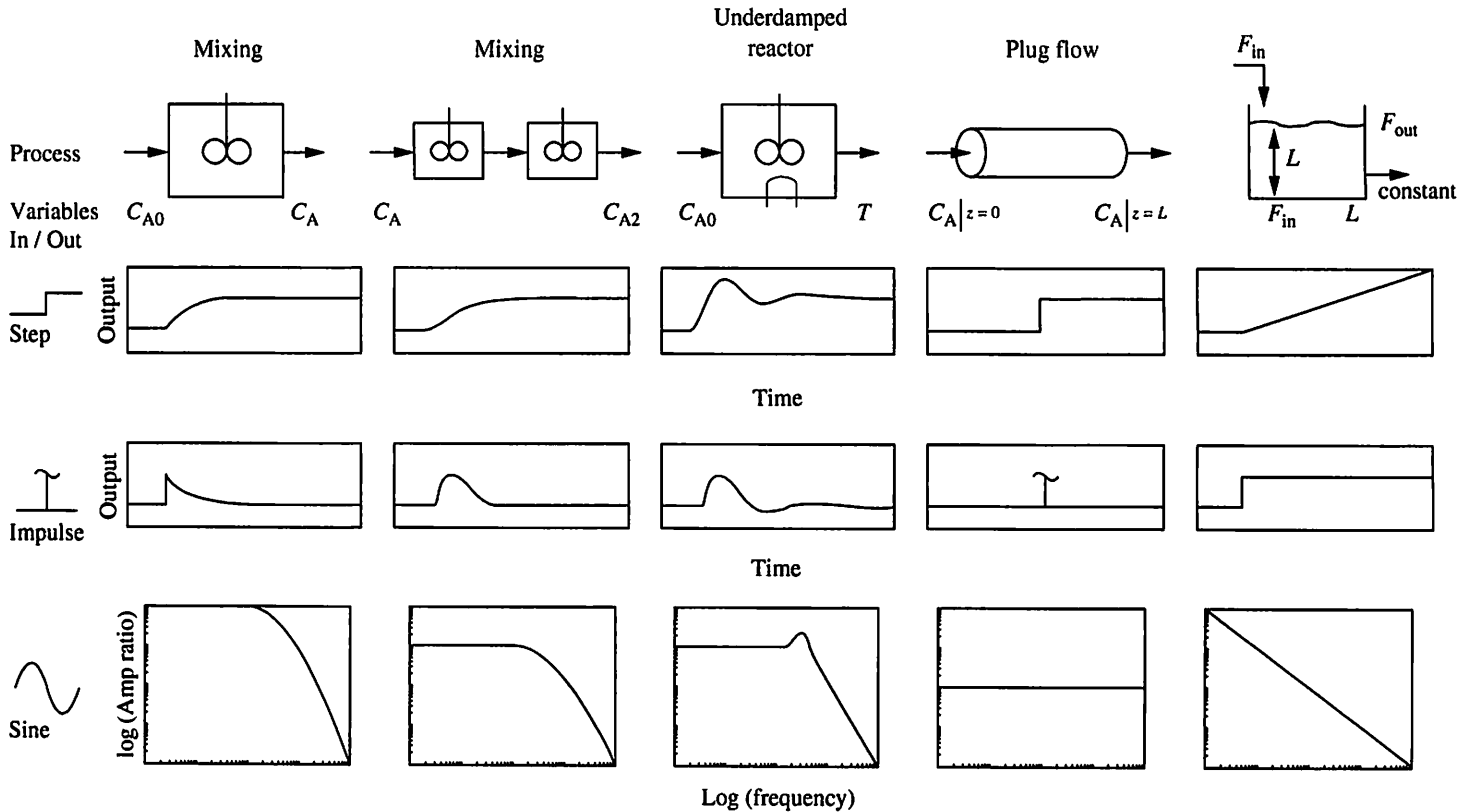
First, the behavior of some simple, basic systems, such as first- and second-order and dead-time systems, is summarized using the results from previous chapters, with some extensions. Second, the behavior of these simple systems in series structures is determined. Third, the behavior of parallel structures of simple systems is introduced. Fourth, the effects of recycle structures on dynamic responses are demonstrated. The chapter concludes with an investigation of more complex physical systems of special importance in the process industries: staged systems and multiple input–multiple output systems.

In these sections, the manner in which the behavior of simple systems is altered by common process structures is derived for simple, idealized models but is demonstrated for important process examples involving levels, heat exchangers,

chemical reactors, and distillation towers. This coverage demonstrates that the engineer must master both the physical principles of specific processes and systems analysis techniques to determine the dynamics of complex processes quantitatively.

## 5.2 ■ BASIC SYSTEM ELEMENTS

The coverage of process dynamics begins with the simplest elements, which are often combined to model more complex systems. Since examples of most of these elements were included in previous chapters, the coverage here is concise. The basic model structure for each element is first defined, and several physical examples are given, with the system input designated by  $X$  and the output by  $Y$ . The chemical process principles should be apparent to the reader, while the electrical and mechanical models are based on Kirchhoff's and Newton's laws, and the reader is referred to Ogata (1992) and Weber (1973) for derivations. The graphical and analytical results of common inputs for several basic systems are summarized in Figure 5.1; the presentation of results in such a figure seems to have originated with Buckley (1964). Only the amplitude ratio is presented here, because more extensive frequency response analysis is presented in Chapter 10, where the importance of the phase behavior on stability is demonstrated and applied in control system analysis.



**FIGURE 5.1**

Dynamic responses for basic process-modelling elements.

## First-Order System

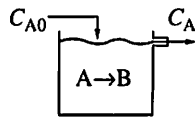
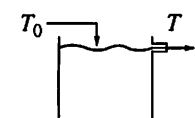
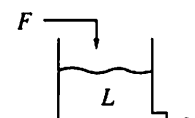
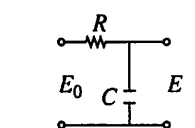
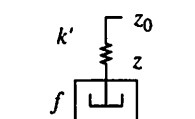
First-order systems occur as the result of a material or energy balance on a lumped (i.e., well-mixed) system, as demonstrated in Examples 3.1 and 3.6. Some further examples are given in Figure 5.2. The differential equation and transfer function for a first-order system are

$$\tau \frac{dY(t)}{dt} + Y(t) = K_p X(t) \quad G(s) = \frac{Y(s)}{X(s)} = \frac{K_p}{\tau s + 1} \quad (5.1)$$

The step response is monotonic, with its maximum slope at the time of the step, and the time to reach 63.2 percent of its final change is one time constant. The final steady-state change is equal to  $K_p(\Delta X)$ .

$$\text{Step response:} \quad Y'(t) = K_p(\Delta X)(1 - e^{-t/\tau}) \quad (5.2)$$

An impulse input occurs over a negligible time and transfers a finite amount into the system. For example, rapidly introducing a small amount of tracer into a stirred tank emulates a perfect impulse. The impulse response shows an immediate increase at the time of the impulse, which for the idealized stirred-tank example would mean that the concentration would change instantly by (mass of tracer)/(volume). After the impulse ( $C$ ), the system follows an exponential path in

	Balance	Input	Output	$K_p$	$\tau$
	Component material	$C_{A0}$	$C_A$	$\frac{F}{F + Vk}$	$\frac{V}{F + Vk}$
	Energy	$T_0$	$T$	1.0	$\frac{V}{F}$
	Overall material	$F$	$L$	$\frac{1}{0.5kL_s^{-0.5}}$	$\frac{A}{0.5kL_s^{-0.5}}$
	Current	$E_0$	$E$	1.0	$RC$
	Force	$z_0$	$z$	1.0	$f/k'$

**FIGURE 5.2**

First-order processes ( $E$  = voltage,  $z$  = position,  $k'$  = spring constant, and  $f$  = friction coefficient).

return to its final condition.

Impulse response: 
$$Y'(t) = \frac{C}{\tau} e^{-t/\tau} \tag{5.3}$$

For the first-order system, the amplitude ratio is never greater than the process gain  $K_p$ , and it decreases monotonically as the frequency increases:

$$AR = |G(j\omega)| = \frac{|Y(j\omega)|}{|X(j\omega)|} = \frac{K_p}{\sqrt{1 + \omega^2\tau^2}} \tag{5.4}$$

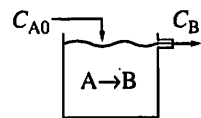
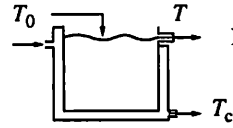
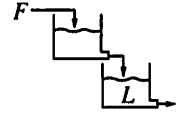
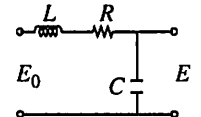
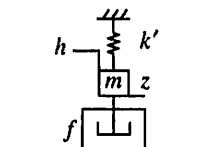
**Second-Order System**

The second-order system occurs when two first-order or one second-order ordinary differential equation is required to model the dynamic behavior. Some examples are given in Figure 5.3. The transfer function for the second-order system with a gain in the numerator (and no zeros) can be written as

$$\tau^2 \frac{d^2Y(t)}{dt^2} + 2\xi\tau \frac{dY(t)}{dt} + Y(t) = K_p X(t) \tag{5.5}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K_p}{\tau^2 s^2 + 2\xi\tau s + 1}$$

with 
$$\alpha_{1,2} = \frac{-\xi \pm \sqrt{\xi^2 - 1}}{\tau}$$

	Balance	Input	Output	$K_p$	$\tau^2$	$2\xi\tau$
	Component material	$C_{A0}$	$C_B$	$\frac{Vk}{F + Vk}$	$\tau_A \tau_B$	$\tau_A + \tau_B$
	Energy	$T_0$	$T$	[see question 5.2]		
	Overall material	$F$	$L$	$\frac{1}{0.5kL_s^{-0.5}}$	$\left[ \frac{A}{0.5kL_s^{-0.5}} \right]^2$	$2\tau$
	Current	$E_0$	$E$	1.0	$LC$	$RC$
	Force	$h$	$z$	$1/k'$	$m/k'$	$f/k'$

**FIGURE 5.3**

Second-order processes ( $E$  = voltage,  $z$  = position,  $k'$  = spring constant,  $f$  = friction coefficient,  $h$  = force,  $m$  = mass,  $\tau_A = V/(F + Vk)$ , and  $\tau_B = V/F$ ).

The parameter  $\xi$  is termed the damping coefficient, and  $\alpha_{1,2}$  are the two roots of the characteristic polynomial, which determine the exponents of the time-domain output function. When the damping coefficient is less than 1.0, the system is termed *underdamped*, the roots of the characteristic polynomial are complex, and the system will have periodic behavior for a nonperiodic input. For example, the nonisothermal reactor system in Section 3.6, which exhibits oscillations for a step input, has a damping coefficient of 0.15. When the damping coefficient is greater than 1.0, the system is termed *overdamped*, the roots of the characteristic polynomial are real, and the system will have nonperiodic responses to nonperiodic inputs. Finally, the series reactor system in Example 3.3 has a damping coefficient of 1.0, which indicates real, repeated roots; this type of system is termed *critically damped*.

Two entries are given in Figure 5.1 for second-order systems; one is for an overdamped system, and the other is for an underdamped system. The step response for the overdamped system initially at steady state is monotonic with an initial slope of zero and an inflection point. Note that the underdamped system experiences periodic behavior even for this simple input.

#### OVERDAMPED STEP RESPONSE ( $\xi > 1$ ).

$$Y = K_p \Delta X \left( 1 + \frac{\tau_1 e^{-t/\tau_1}}{\tau_2 - \tau_1} - \frac{\tau_2 e^{-t/\tau_2}}{\tau_2 - \tau_1} \right) \quad (5.6)$$

#### CRITICALLY DAMPED STEP RESPONSE ( $\xi = 1$ ).

$$Y = K_p \Delta X \left[ 1 - \left( 1 + \frac{t}{\tau} \right) e^{-t/\tau} \right] \quad (5.7)$$

#### UNDERDAMPED STEP RESPONSE ( $\xi < 1$ ).

$$Y = K_p \Delta X - K_p \frac{\Delta X}{\sqrt{1 - \xi^2}} e^{-\xi t/\tau} \sin \left( \frac{\sqrt{1 - \xi^2}}{\tau} t + \phi \right) \quad (5.8)$$

$$\phi = \tan^{-1} \left( \frac{\sqrt{1 - \xi^2}}{\xi} \right)$$

#### OVERDAMPED IMPULSE RESPONSE ( $\xi > 1$ ).

$$Y = C \left( \frac{e^{-t/\tau_1}}{\tau_1 - \tau_2} - \frac{e^{-t/\tau_2}}{\tau_1 - \tau_2} \right) \quad (5.9)$$

#### CRITICALLY DAMPED IMPULSE RESPONSE ( $\xi = 1$ ).

$$Y = \frac{Ct}{\tau^2} e^{-t/\tau} \quad (5.10)$$

### UNDERDAMPED IMPULSE RESPONSE ( $\xi < 1$ ).

$$Y = \frac{C}{\tau\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin\left(\frac{\sqrt{1-\xi^2}}{\tau} t\right) \quad (5.11)$$

Both the step and impulse responses for a second-order system have initial responses that are more gradual than for a first-order system. The overdamped system approaches its final value smoothly, while the underdamped system experiences oscillations.

The amplitude ratio of the frequency response is monotonically decreasing for an overdamped system and begins to deviate substantially from  $K_p$  around the frequency equal to  $1/\tau$ . The amplitude ratio for second-order systems with a damping coefficient below 0.707 exceeds  $K_p$  over a limited frequency range around  $1/\tau$ . This resonance effect results from the inherent oscillatory tendency of the system reinforcing the input sine oscillations.

$$\text{AR} = |G(j\omega)| = \frac{|Y(j\omega)|}{|X(j\omega)|} = \frac{K_p}{\sqrt{(1-\omega^2\tau^2)^2 + (2\omega\tau\xi)^2}} \quad (5.12)$$

### Dead Time

The dead time or transportation delay was introduced in Example 4.3 for plug flow of liquids and can also occur for transportation of solids along a conveyor belt. It was shown to have the following model:

$$Y(t) = X(t - \theta) \quad G(s) = \frac{Y(s)}{X(s)} = e^{-\theta s} \quad (5.13)$$

The step response, impulse response, and amplitude ratio can all be easily determined, because the output is the input translated in time by  $\theta$ . For example, this leads to the conclusion that the amplitude ratio is equal to 1.0 for all frequencies, which can be demonstrated mathematically by

$$\text{AR} = |e^{-j\omega\theta}| = |\cos(\omega\theta) - j \sin(\omega\theta)| = \sqrt{\cos^2(\omega\theta) + \sin^2(\omega\theta)} = 1 \quad (5.14)$$

The dead time can be *approximated* by a transfer function that replaces the exponential in the Laplace variable ( $e^{-\theta s}$ ) with a ratio of polynomials in  $s$ . This approach is referred to as a Pade approximation, which is presented in Appendix D. In this book, we will not use dead time approximations; i.e., we will model the dead time as an exact delay as given in equations (5.13).

The importance of dead time to feedback control can be understood by considering an example such as steering an automobile. With dead time, the automobile would not respond immediately after the change in steering wheel position. Clearly, such an automobile would be difficult to drive and would require a skilled and patient driver who could wait for the effect of a steering wheel change to occur.

### Integrator

The integrator is a special type of first-order system; a process example of an integrator is a level system, which is modelled based on an overall material balance

to give

$$\rho A \frac{dL}{dt} = \rho F_0 - \rho F_1 \quad (5.15)$$

In many cases the inlet and outlet flows do not depend on the level (unlike the tank draining Example 3.6). When no causal relationship exists from the level to the flow, the model has the following general form:

$$\tau_H \frac{dY'}{dt} = X' \neq f(Y') \quad \tau_H = \text{holdup time} \quad (5.16)$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{\tau_H s} \quad (5.17)$$

The important difference between the integrator and the first-order system in equation (5.1) is the lack of dependence of the derivative on the output variable ( $Y'$ ); that is,  $dY'/dt$  is independent of  $Y'$ . This results in a pole at  $s = 0$  in the transfer function. The analytical expression for the output of the integrator is

$$Y'(t) = \int_0^t X'(t') dt' \quad (5.18)$$

A system like this simply accumulates the net input: thus, the name *integrator*. If the deviation in the input remains nonzero and of the same sign, the magnitude of the idealized model output increases without limit as time increases toward infinity. For a step input,

$$\text{Step response:} \quad Y' = \frac{\Delta X}{\tau_H} t \quad (5.19)$$

The impulse response also demonstrates that the system integrates the impulse (area under the impulse function), and then the output remains constant at its altered value when  $X'(t)$  returns to zero. The value of the impulse response is  $Y' = C/\tau_H$ .

The amplitude ratio can be determined to be

$$\text{Frequency response:} \quad \text{AR} = |G(j\omega)| = \left| \frac{1}{\tau_H j\omega} \right| = \left| \frac{-\omega j}{\tau_H \omega^2} \right| = \frac{1}{\tau_H \omega} \quad (5.20)$$

As the frequency decreases, the amount accumulated by the integrator each half period (which is related to the output amplitude) increases.

## Self-Regulation

The unique behavior of the integrator demonstrates that not all processes tend to a steady state after input changes cease and all inputs are constant. To clarify the distinction, the term *self-regulation* is introduced here.

For a process that is self-regulatory, the output variables tend to a steady state after the input variables have reached constant values.

Many processes encountered to this point have been self-regulatory, including the chemical reactors, heat exchanger, and mixing tanks. Self-regulatory processes are

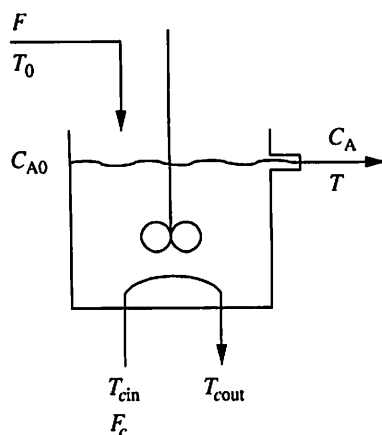
generally easier to operate because they tend to a steady state. Naturally, the final steady state might be acceptable or not depending on the magnitude and direction of the input changes, so that process control is often applied to self-regulatory processes.

The self-regulation in a process can be identified by analyzing the dynamic model to determine if the value of the output variable influences its derivative. For example, the heat exchanger in Example 3.7 has inherent negative feedback, because an increase in the output (outlet temperature) causes a decrease in a model input term  $-(F/V + UA/V\rho C_p)T$ , which stabilizes the system by decreasing the derivative:

$$\frac{dT}{dt} = \underbrace{\left(\frac{F}{V}T_0 + \frac{UA}{V\rho C_p}T_{cin}\right)}_{\text{External inputs}} - \underbrace{\left(\frac{F}{V} + \frac{UA}{V\rho C_p}\right)T}_{\text{Inherent negative feedback}} \quad (5.21)$$

Some processes have inherent *positive* and negative feedback; for example, the nonisothermal chemical reactor with exothermic chemical reaction is

$$\begin{aligned} \frac{dT}{dt} = & \underbrace{\left(\frac{F}{V}T_0 + \frac{UA}{V\rho C_p}T_{cin}\right)}_{\text{External inputs}} - \underbrace{\left(\frac{F}{V} + \frac{UA}{V\rho C_p}\right)T}_{\text{Inherent negative feedback}} \\ & + \underbrace{\frac{(-\Delta H_{rxn})k_0e^{-E/RT}C_A}{\rho C_p}}_{\text{Inherent positive feedback}} \end{aligned} \quad (5.22)$$



The reactor has a negative feedback term in its energy balance, the same as for the heat exchanger. However, the exothermic chemical reaction contributes positive feedback, because the input term  $(-\Delta H_{rxn}k_0e^{-E/RT}C_A/\rho C_p)$  increases when the output temperature increases. For the parameter values in Table C.1, case I, the inherent negative feedback in the process dominates, and the process achieves a steady state after a step input. The positive feedback is substantial, however, which leads to the periodic behavior and complex poles. Additional comments on the behavior and stability of processes are given in Appendix C.

In contrast, *non-self-regulatory processes* do not tend to steady-state operation after all inputs have reached constant values. Thus, even a small (and constant) input change from an initial steady state can lead to large disturbances after a long time. A non-self-regulatory process can be identified from its dynamic model; the value of the output variable does not influence its derivative, as shown in equation (5.15), so that the derivative can have a constant (nonzero) value over a long time. Without intervention, a non-self-regulatory process can experience very large deviations from desired values; therefore, all non-self-regulatory processes require process control. The dynamics of typical non-self-regulatory processes are covered in Chapter 18, along control technology tailored to their special requirements.

In summary, many different systems obeying the models of these basic elements behave in a similar manner. After the parameters have been determined, their behavior for specified inputs is well understood. Thus, the experience learned from a few examples can be extended, with care, to many other systems.



### 5.3 □ SERIES STRUCTURES OF SIMPLE SYSTEMS

A structure involving a series of systems occurs often in process control. As discussed in Chapter 2, this structure can occur because of a processing sequence—for example, feed heat exchange, chemical reactor, product cooling, and product separation. Also, a control loop involves a final element (valve), process, and sensor in a series, as will be more fully discussed in Part III. Therefore, the understanding of how series structures behave is essential in the design of chemical plants and process control systems.

#### Noninteracting Series

There are two major categories of series systems, and the noninteracting system is covered first. It is worthwhile considering the mixing system, which conforms to the block diagram at the bottom of Figure 5.4a, in which each intermediate variable has physical meaning.

$$V \frac{dC'_{A1}}{dt} = FC'_{A0} - FC'_{A1} \quad (5.23)$$

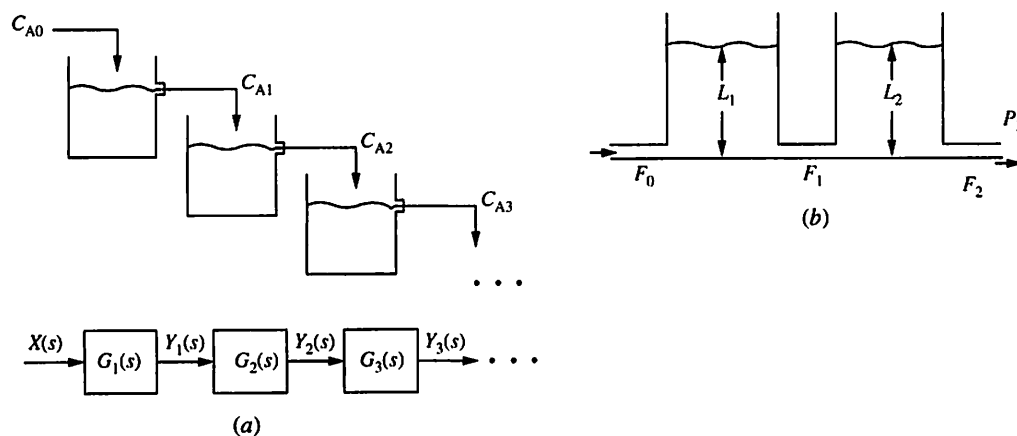
$$V \frac{dC'_{A2}}{dt} = FC'_{A1} - FC'_{A2} \quad (5.24)$$

Note that the model equations have the general form

$$\tau_i \frac{dY'_i}{dt} = K_i Y'_{i-1} - Y'_i \quad \text{for } i = 1, \dots, n \quad \text{with } Y'_0 = X' \quad (5.25)$$

Any system modelled with equations of this structure constitutes a noninteracting series system. Important features of the system follow from this model.

1. Only  $Y_{n-1}$  and  $Y_n$  (not  $Y_{n+1}$ ) appear in the equation for  $dY_n/dt$ .
2. Following from (1), the downstream properties do not affect upstream properties; in the example, the concentration in tank 2 does not affect the concentration in tank 1 but does affect tank 3.



**FIGURE 5.4**

Series of processes: (a) noninteracting; (b) interacting.

3. The model for the general noninteracting series of first-order systems can be developed by taking the Laplace transform of each equation (5.25) and combining them into one input-output expression. For a series of systems shown in Figure 5.4a, each represented by a transfer function  $G_i(s)$ , the overall transfer function

$$\frac{Y_n(s)}{X(s)} = G_n(s)G_{n-1}(s) \cdots G_1(s) = \prod_{i=0}^{n-1} G_{n-i}(s) \quad (5.26)$$

For  $n$  first-order systems in series, this gives

$$\frac{Y_n(s)}{X(s)} = \frac{\prod_{i=0}^{n-1} K_{n-i}}{\prod_{i=0}^{n-1} (\tau_{n-i}s + 1)} \quad \text{with } K_{n-i} \text{ and } \tau_{n-i} \text{ for the individual systems} \quad (5.27)$$

The gains and time constants appearing in equation (5.27) are the same as the values for the individual systems, as in equation (5.25). Thus, the model of interacting systems can be determined directly from the individual models.

4. If each system is stable (i.e.,  $\tau_i > 0$  for all  $i$ ), the series system is stable. This follows from the important observation that the poles (roots of the characteristic polynomial) of the series system are the poles of the individual systems.

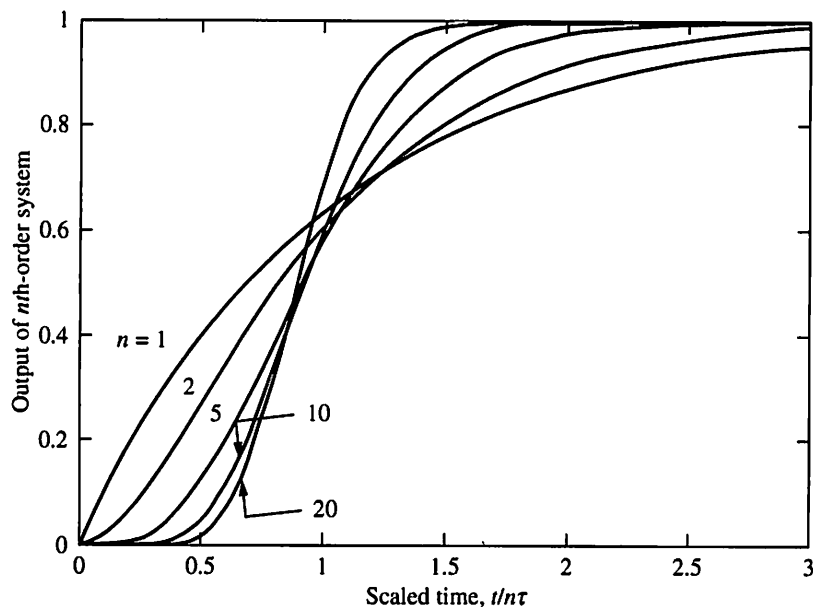
Now the dynamic response of a series of noninteracting first-order systems can be considered. Since so many possibilities exist, the simplest case of  $n$  identical systems, all with unity gain, is considered. The response to a step in the input,  $X'(s) = 1/s$ , is plotted in Figure 5.5. Note that the time is divided by the order of the system (i.e., the number of systems in series), which time-scales the responses for easy comparison. We note that the shape of the response changes from the now-familiar exponential curve for  $n = 1$ . As  $n$  increases, the response begins to have an apparent dead time, which is the result of several first-order systems in series. For very large  $n$ , the output response has a very steep change at time equal to  $n\tau$ . Thus, we conclude that the series of identical noninteracting first-order systems approaches the behavior of a dead time with  $\theta \approx n\tau$  for large  $n$ . Again looking ahead to feedback control, a system with several first-order systems in series would seem to be difficult to control, for the same reasons discussed for dead times.

A second observation is that the curves all reach 63 percent of their output change at approximately the same value of  $t/n\tau$ ; this will be exploited later in the section. Finally, we note that the system is always overdamped, because the transfer function has  $n$  real poles, all at  $-1/\tau$ .

The amplitude ratio of the frequency response can be determined directly from the transfer function in equation (5.27) to be

$$\text{AR} = \frac{|Y_n(j\omega)|}{|X(j\omega)|} = |G(j\omega)| = \left( \prod_{i=1}^n K_i \right) \left( \frac{1}{\sqrt{1 + \omega^2 \tau^2}} \right)^n \quad (5.28)$$

The amplitude ratio is always less than or equal to the overall gain, and it decreases rapidly as the frequency becomes large. Amplitude ratios for several series of



**FIGURE 5.5**

Responses of  $n$  identical noninteracting first-order systems with  $K = 1$  in series to a unit step at  $t = 0$ .

identical first-order systems are shown in Figure 5.6; again, the frequency is scaled to the order of the system to provide time-scaling.

### Interacting Series

The second major category of series systems is interacting systems. Again, it is worthwhile considering a physical example, this being the level-flow process in Figure 5.4*b*. Assuming that the flow through each pipe is a function of the pressure difference, the model can be derived based on overall material balance for each vessel to give

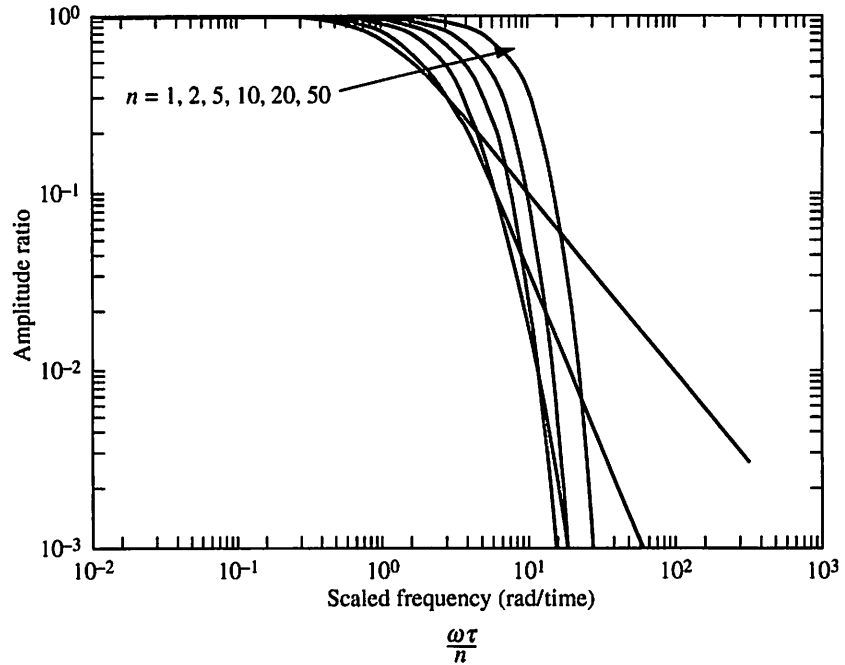
$$\begin{aligned} A_i \frac{dL_i}{dt} &= F_{i-1} - F_i \\ &= K_{i-1}(L_{i-1} - L_i) - K_i(L_i - L_{i+1}) \end{aligned} \quad (5.29)$$

because  $F_i = K'_i(P_i - P_{i+1})$  for the linearized system, and the pressures are proportional to the liquid levels. These model equations have the following general form for a series of two interacting first-order systems:

$$H_1 \frac{dY'_1}{dt} = X' - K_1(Y'_1 - Y'_2) \quad (5.30)$$

$$H_2 \frac{dY'_2}{dt} = K_1(Y'_1 - Y'_2) - K_2(Y'_2 - Y'_3) \quad (5.31)$$

Many important physical systems, including that in Figure 5.4*b*, have structures described by equations (5.30) and (5.31); thus, these equations are considered representative of interacting systems for subsequent analysis. Some important features of these systems follow from their model structure:



**FIGURE 5.6**

Frequency responses of  $n$  identical noninteracting first-order systems with  $K = 1$  in series.

1. The variables  $Y_{n-1}$ ,  $Y_n$ , and  $Y_{n+1}$  appear in the equation for  $dY_n/dt$ .
2. Following from (1), the downstream properties affect upstream properties; for example, the exhaust pressure ( $P_3$ ) influences *both* levels in Figure 5.4b.
3. The model for the general interacting series system of first-order systems can be developed by taking the Laplace transform of equations (5.30) and (5.31) and combining them into one input-output expression, which results in poles of the interacting system that are *different* from the poles of the individual systems.

The procedure for deriving the overall transfer function is shown in some detail, because the result is somewhat more complex than for a noninteracting system and because the procedure can be applied to systems of differing structures. First, the Laplace transform of equation (5.30) can be rearranged to give (with the primes deleted)

$$Y_1(s) = \frac{1/K_1}{\tau_{Y1}s + 1} X(s) + \frac{1}{\tau_{Y1}s + 1} Y_2(s) \quad \text{with } \tau_{Y1} = \frac{H_1}{K_1} \quad (5.32)$$

The parameter  $\tau_{Y1}$  is the time constant for the first system when considered *individually*. The Laplace transform of the second equation is

$$\tau_{Y2}s Y_2(s) = \frac{K_1}{K_2} [Y_1(s) - Y_2(s)] - [Y_2(s) - Y_3(s)] \quad \text{with } \tau_{Y2} = \frac{H_2}{K_2} \quad (5.33)$$

Again, the parameter  $\tau_{Y2}$  is the time constant for the second system when considered *individually*. The behavior of the combined system can be determined by

substituting equation (5.32) into (5.33) to give, after some rearrangement,

$$Y_2(s) = \frac{(\tau_{Y1}s + 1)}{\tau_{Y1}\tau_{Y2}s^2 + \left(\tau_{Y1} + \tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2}\right)s + 1} Y_3(s) + \frac{1/K_2}{\tau_{Y1}\tau_{Y2}s^2 + \left(\tau_{Y1} + \tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2}\right)s + 1} X(s) \quad (5.34)$$

Several important conclusions on the effect of the series structure on the dynamic behavior can be determined from an analysis of the denominator of the transfer function. The time constants of the interacting system ( $\tau_1$  and  $\tau_2$ ), which are the inverses of the poles, can be determined by solving the quadratic equation for the roots of the characteristic polynomial to give

$$\alpha_{1,2} = \frac{1}{\tau_{1,2}} = \frac{-\left(\tau_{Y1} + \tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2}\right) \pm \sqrt{\left(\tau_{Y1} + \tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2}\right)^2 - 4\tau_{Y1}\tau_{Y2}}}{2\tau_{Y1}\tau_{Y2}} \quad (5.35)$$

Four characteristics of the dynamics of this type of series system are now established. First, the possibility of complex poles is determined to establish whether periodic behavior is possible. The expression within the square root in equation (5.35) can be rearranged to give

$$\begin{aligned} & \left(\tau_{Y1} + \tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2}\right)^2 - 4\tau_{Y1}\tau_{Y2} \\ & = (\tau_{Y1} - \tau_{Y2})^2 + \tau_{Y1}\frac{K_1}{K_2} \left(2\tau_{Y1} + 2\tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2}\right) > 0 \end{aligned} \quad (5.36)$$

Since both terms in the right-hand expression are greater than zero, the entire expression is greater than zero, and complex poles are not possible for this system. Therefore, periodic behavior cannot occur for nonperiodic inputs, such as a step.

Second, the stability of the process can be determined from equation (5.35). Note that the numerator has the form  $-a \pm (a^2 - b)^{0.5}$ , with  $a$  and  $b$  both positive. Therefore, the poles for both signs of the root are negative, and the system is stable.

Third, the "speed" of response of the interacting series system can be compared with the individual system responses. Since the poles are real, the characteristic polynomial in equation (5.34) can be written in an equivalent form as

$$(\tau_1s + 1)(\tau_2s + 1) = \tau_1\tau_2s^2 + (\tau_1 + \tau_2)s + 1 \quad (5.37)$$

Equating the coefficients of like powers of  $s$  in equations (5.34) and (5.37) gives

$$\tau_1\tau_2 = \tau_{Y1}\tau_{Y2} \quad \text{and} \quad \tau_1 + \tau_2 = \tau_{Y1} + \tau_{Y2} + \tau_{Y1}\frac{K_1}{K_2} \quad (5.38)$$

Therefore, the sum of the time constants for the overall interacting system is greater than the sum of the individual systems. In other words, the interacting system is "slower," due to the interaction, than it would have been if the systems were noninteracting.

Fourth, equations (5.38) show that the product of the time constants is unchanged but the sum is greater. Therefore, the difference between the interacting system time constants ( $\tau_1 - \tau_2$ ) is greater than the difference between the individual time constants ( $\tau_{Y1} - \tau_{Y2}$ ); that is, one time constant begins to dominate. This conclusion can be demonstrated by rearranging equations (5.38) to give

$$(\tau_1 - \tau_2)^2 = (\tau_{Y1} - \tau_{Y2})^2 + \tau_{Y1} \frac{K_1}{K_2} \left( 2\tau_{Y1} + 2\tau_{Y2} + \tau_{Y1} \frac{K_1}{K_2} \right) \quad (5.39)$$

Since the noninteracting series system has been shown to have all real poles, the dynamic responses of an interacting system of first-order systems have many of the same characteristics as those of a noninteracting system; that is, they are stable and overdamped.

The previous results for interacting systems are applicable to (only) those systems that conform to the model; in addition to having variables  $Y_{n-1}$ ,  $Y_n$ , and  $Y_{n+1}$  appear in the equation for  $dY_n/dt$ , the coefficients of each linearized term must conform to the structure and range of values in equations (5.30) and (5.31).

Many systems have the same model structures but different ranges for the values of the parameters. If the type of system is not obvious from the structure of the equations and the values of the model parameters, the model can be analyzed using the procedure just applied to the equations (5.30) and (5.31) to determine important characteristics of its dynamic behavior.

### Noninteracting Series with Dead Time

As will become more apparent in the next chapter, we often use first-order-with-dead-time models to approximate more complex systems with monotonic step input responses. Therefore, noninteracting series of first-order-with-dead-time systems are considered to conclude this section. The direct application of equation (5.26) results in

$$\frac{Y(s)}{X(s)} = \prod_{i=0}^{n-1} G_{n-i}(s) = \frac{\left( \prod_{i=1}^n K_i \right) \exp \left( - \sum_{i=1}^n \theta_i s \right)}{\prod_{i=1}^n (\tau_i s + 1)} \quad \text{with } G_i(s) = \frac{K_i e^{-\theta_i s}}{\tau_i s + 1} \quad (5.40)$$

This overall transfer function provides the basis for the following equations, which give values for key parameters of a noninteracting series of first-order-with-dead-time systems.

Exact relationships:  $K = \prod_{i=1}^n K_i \quad \theta = \sum_{i=1}^n \theta_i \quad (5.41a)$

Approximate relationship:  $t_{63\%} \approx \sum_{i=1}^n (\theta_i + \tau_i) \quad (5.41b)$

The results for the overall gain and dead time follow directly from equation (5.40). The approximation for the time for the output response to a step input to reach 63 percent of its final value,  $t_{63\%}$ , is based on fitting an approximate model to the response of the series system, using the method of moments. The derivation of this expression is provided in Appendix D. The relationships in equations (5.41) are useful for quickly characterizing the approximate behavior of a noninteracting series system from the individual systems; comparison to solutions of noninteracting systems (e.g., Figure 5.5) shows that the expression for  $t_{63\%}$  is a reasonable approximation but *not exact*.

#### EXAMPLE 5.1.

Four first-order-with-dead time systems, with parameters in the following table, are placed in a noninteracting series. Describe the output response of this system to a step change in the input to the series at time = 2.

System	1	2	3	4
Dead time, $\theta$	0.40	0.90	1.2	1.70
Time constant, $\tau$	1.5	3.3	5.2	0.95
Gain, $K$	1.0	0.25	3.0	1.33

The results in this section on noninteracting systems indicate that the output response will be an overdamped sigmoid. Equations (5.41) can be used to estimate key values of the response. Note that the input occurred at time = 2, so that the points indicated on Figure 5.7 are based on the following results as measured from time = 2.

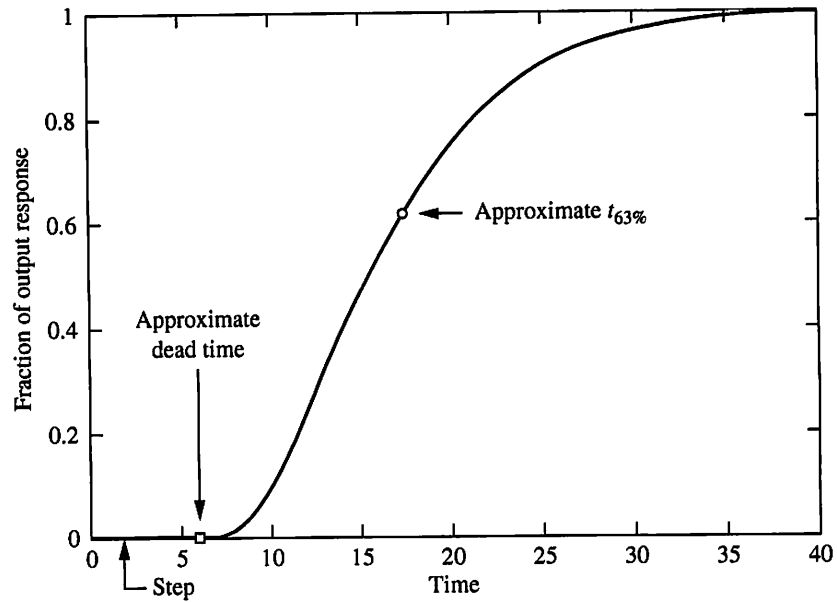
$$K_p = 1.0 \quad \theta = 4.2 \text{ (after step)} \quad \sum(\theta + \tau) = 15.15 \quad \therefore t_{63\%} \approx 15.15 \text{ (after step)}$$

The overall response is compared with the approximation in Figure 5.7, which demonstrates the usefulness of the approximation for  $t_{63\%}$ , because it gives an approximate "time scale" for the response. However, many sigmoidal curves could be drawn through the two points in the figure. The entire curve can be determined through analytical or numerical solution of the defining equations.

#### EXAMPLE 5.2. Input-output response.

Two series systems, each with four elements, involve only transportation delays and mixing tanks. A step change is introduced into the input feed composition of each system with the flow rates constant. Determine and compare the dynamic responses of the output for each system. Since there is no chemical reaction, the systems have a gain of 1.0 and dynamic parameters given in the following table.

	$\theta_1$	$\tau_1$	$\theta_2$	$\tau_2$	$\theta_3$	$\tau_3$	$\theta_4$	$\tau_4$
Case 1	0	2	2	0	0	2	2	0
Case 2	0	2	2	2	1	0	1	0



**FIGURE 5.7**

**Dynamic response of series processes in Example 5.1 for a unit step at time = 2.**

The solution can be developed in several ways. The most general is to derive the overall input-output transfer functions for these systems.

$$\begin{aligned}
 Y_4(s) &= G_4(s)Y_3(s) = \cdots = G_4(s)G_3(s)G_2(s)G_1(s)X(s) \\
 \frac{Y_4(s)}{X(s)} &= \frac{1.0e^{-(\theta_1+\theta_2+\theta_3+\theta_4)s}}{(\tau_1s+1)(\tau_2s+1)(\tau_3s+1)(\tau_4s+1)} \\
 &= \frac{1.0e^{-4s}}{(2s+1)(2s+1)}
 \end{aligned}$$

Since the overall transfer functions are the same for the two systems, their dynamic input-output behaviors are *identical*. This is verified by the transient responses of the two cases for a step input at time = 2 in Figure 5.8, with each variable  $Y_i(t)$  on a separate scale.

The responses in Figure 5.8 show that two systems can have the same input-output behavior with different values for intermediate variables.

In conclusion, the analysis in this section has demonstrated that both noninteracting and interacting series of  $n$  first-order systems can be modelled by a transfer function with a characteristic polynomial of order  $n$ . Much about the dynamic responses of the series systems can be determined from the models of the individual systems. The results are summarized in Table 5.1.

The series systems in this section provided additional reinforcement for the importance of transfer function poles. The strongest general conclusions were based



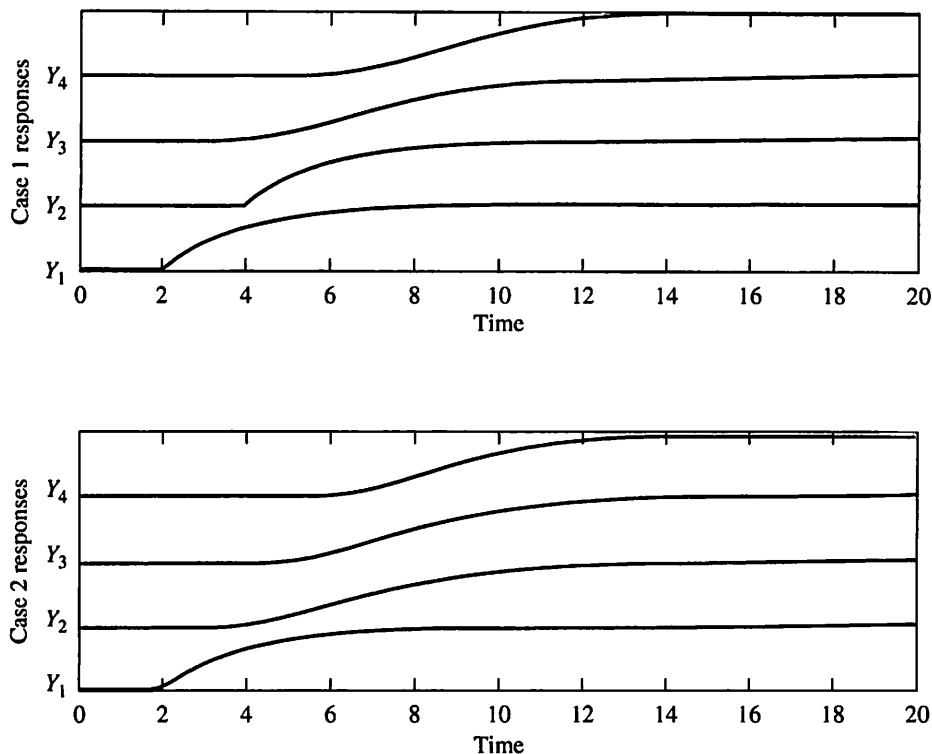


FIGURE 5.8

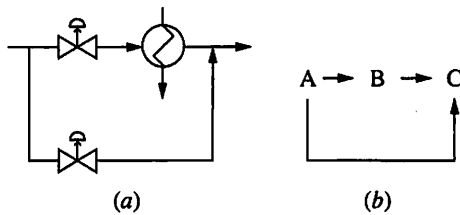
Dynamic responses for series system in Example 5.2 to a unit step at time = 2.

TABLE 5.1

**Properties of series systems with first-order elements  
(responses between input,  $X$ , and output,  $Y_n$ )**

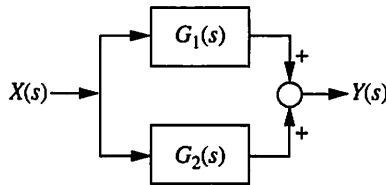
Individual first-order systems	Noninteracting series systems	Interacting series system, equations (5.30) and (5.31)
$n$ first-order systems	$n$ th-order system	$n$ th-order system
Each is stable	Stable, not periodic	Stable, not periodic
Time constants, $\tau_i$	Time constants are $\tau_i$ , $i = 1, \dots, n$	Time constants are not $\tau_i$ 's. They must be determined by solving the characteristic polynomial.
$t_{63\%}$	$t_{63\%} \approx \sum \tau_i$	$t_{63\%} > \sum \tau_i$
Step response	Overdamped, sigmoidal	Overdamped, sigmoidal
Frequency response	$AR \leq K_p$ for all $\omega$	$AR \leq K_p$ for all $\omega$

on the manner in which the poles of the overall system were or were not affected by the series structure. These conclusions concerned stability and the related property of periodic behavior. Since these generalizations dealt with properties completely determined by the poles, they are independent of the numerators in the transfer functions. In fact, the generalizations on stability and periodicity can be extended to any series transfer functions with denominators expressed as a polynomial in  $s$ .



**FIGURE 5.9**

Examples of parallel systems in chemical engineering: (a) heat exchanger with bypass and (b) chemical reaction system.



**FIGURE 5.10**

Example of a parallel structure involving two systems.

However, the values of the poles do not provide general conclusions for the time-domain responses to step and sine inputs. Since both the numerator and denominator of the transfer function influence the dynamic behavior, the more specific results on dynamic responses are valid only for systems consistent with the assumptions in the derivations—that is, with a constant for the numerator of each series transfer function element. In particular, Figures 5.4 and 5.5 and all conclusions on the step response and amplitude ratio are specific to systems whose component elements have constant numerators. Finally, such strong conclusions for an overall system, based on the individual elements, are not always possible, as demonstrated by the structures considered in the remainder of this chapter.

## 5.4 ■ PARALLEL STRUCTURES OF SIMPLE SYSTEMS

Parallel paths between a system input and its output can occur in processes, for example, the heat exchanger with multiple fluid flow paths in Figure 5.9a and the multiple reaction pathways in Figure 5.9b. Systems with parallel paths can experience unique dynamic behavior that can have a strong effect on control performance. Therefore, engineers should understand the process structures leading to parallel structures giving good and poor dynamic behaviors. The basic concepts of parallel systems are introduced in this section to explain the reasons for the unique dynamic behavior, and detailed process examples are presented in Appendix I.

A simple structure that demonstrates the important features of parallel systems is shown in Figure 5.10. The system has two paths between the input variable,  $X$ , and the output,  $Y$ . The overall model relating input and output can be determined using block diagram algebra.

$$Y_1(s) = G_1(s)X(s) \quad (5.42)$$

$$Y_2(s) = G_2(s)X(s) \quad (5.43)$$

$$Y(s) = Y_1(s) + Y_2(s) \quad (5.44)$$

The three equations can be combined to give

$$\frac{Y(s)}{X(s)} = G_1(s) + G_2(s) \quad (5.45)$$

For the situation in which each process is a first-order process,  $G_i(s) = K_i/(\tau_i s + 1)$ , the model becomes

$$\frac{Y(s)}{X(s)} = \frac{K_1}{(\tau_1 s + 1)} + \frac{K_2}{(\tau_2 s + 1)} \quad (5.46)$$

Equation (5.46) can be rearranged to have a common denominator to give

$$\frac{Y(s)}{X(s)} = \frac{K_p(\tau_3 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (5.47)$$

$$\begin{aligned} \text{with } K_p &= (K_1 + K_2) \\ \tau_3 &= (K_1 \tau_2 + K_2 \tau_1)/(K_1 + K_2) \end{aligned}$$

We note that the transfer function model in equation (5.47) has a polynomial in the Laplace variable  $s$  in the denominator, as has occurred in many previous models; the denominator terms result from taking the Laplace transform of derivatives in

the dynamic models. Since the stability and periodicity of the output  $Y(t)$  depend on the roots of the denominator, we conclude that the parallel structure does not alter these important aspects of dynamic behavior.

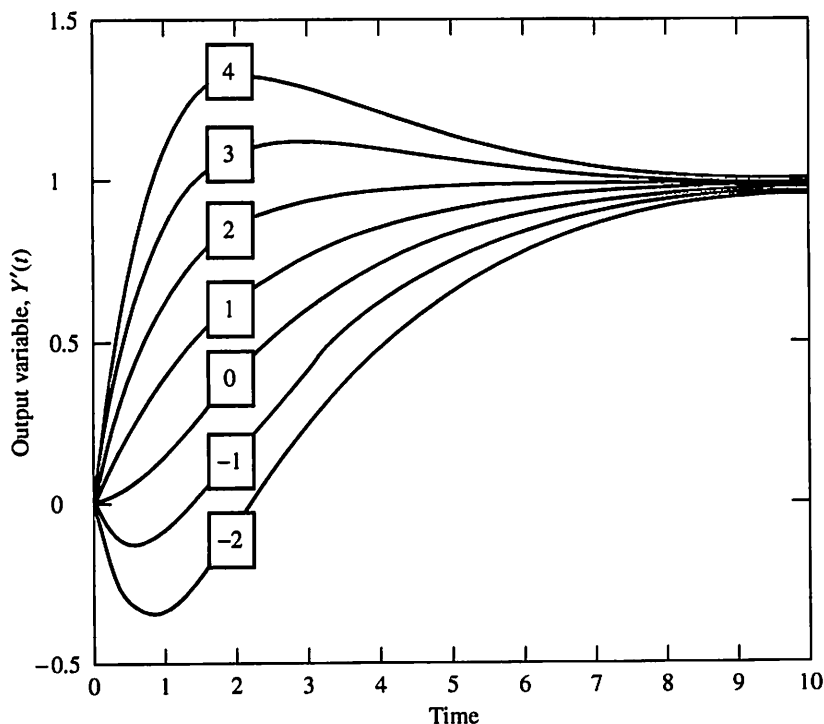
In addition, this model has a new feature in the model, a polynomial in  $s$  in the transfer function numerator that results from the parallel structure. To investigate the effect of the parallel structure on dynamic behavior, the step response of the system in Figure 5.10 and modelled by equation (5.47) will be determined. The time behavior can be determined by setting  $X(s) = \Delta X/s$  for a step change and taking the inverse Laplace transform using entry 10 in Table 4.1 (with  $a = \tau_3$ ).

$$Y'(t) = K_p \Delta X \left( 1 + \frac{\tau_1 - \tau_3}{\tau_2 - \tau_1} e^{-t/\tau_1} - \frac{\tau_2 - \tau_3}{\tau_2 - \tau_1} e^{-t/\tau_2} \right) \quad (5.48)$$

To enable us to plot a typical system, the following arbitrary parameter values are inserted into equation (5.48):  $K = 1$ ,  $\Delta X = 1$ ,  $\tau_1 = 2$ , and  $\tau_2 = 1$ . The responses are plotted for several values of the parameter  $\tau_3$  in Figure 5.11.

Key characteristics of the responses depend on the value of  $\tau_3$ .

For negative values of  $\tau_3$  the step response changes initially in the *direction opposite* from the final steady state! This behavior is termed an *inverse response* and results from the parallel path.



**FIGURE 5.11**

Responses for a sample parallel system to a unit step at  $t = 0$  in  $X(s)$ ; the model is  $Y(s)/X(s) = G(s) = (\tau_3 s + 1)/(2s + 1)(s + 1)$ , with the value of  $\tau_3$  shown for each curve.

This behavior can be explained by considering the system in Figure 5.10, which shows that the output is the sum of two effects. When one path has fast dynamics and a negative gain, the process output initially decreases; however, if the second path has slower dynamics but a positive gain of larger magnitude, the ultimate output response will be positive. Thus, an inverse response occurs.

Figure 5.11 also shows that the output can have transient values greater than its final value when  $\tau_3 > \tau_1$  and  $\tau_3 > \tau_2$ . This behavior is termed *overshoot* and results from the parallel path. This behavior can be explained by considering the system in Figure 5.10. When one path has fast dynamics and a large positive gain, the process output initially increases a large amount; when the effects of the second slower path are negative but smaller in magnitude, the output decreases from its maximum, but remains positive. Thus, the overshoot occurs although the process is overdamped, i.e., nonperiodic.

The importance of inverse response or overshoot can be recognized by thinking about how you would drive an automobile that had steering dynamics with either of these behaviors. Only a skilled driver could maintain the vehicle on the road, and no driver could achieve good performance. Therefore, the design engineer should seek to avoid processes that experience these behaviors through process equipment selection. Note that the dynamics are monotonic for many systems in Figure 5.11 when  $\tau_3 \neq 0$ , so that only parallel structures with specific ranges of parameters yield these unique and usually undesirable behaviors. In Appendix I, some realistic parallel-path process examples are presented that experience interesting and important dynamic behavior. Approaches to improve dynamic performance through control are discussed throughout the book.

In summary, parallel paths exist in many processes due to either complex interconnecting flow structures of individual systems or due to parallel effects within a single process. Since the poles are unaffected by a parallel structure, stability and damping of the overall system is not affected. This can be seen from equation (5.47), in which the denominator of the overall transfer function has the poles of the individual transfer functions. However, the parallel paths can have a significant effect on the dynamic behavior of the system, and the most complex behavior—overshoot or inverse response—occurs when parallel paths have significantly *different speeds of response*, so that parallel responses from an input affect the output at different times. Also, the approximate time to reach 63 percent of the output change for a step input is affected by the numerator, and it is not simply the sum of the individual time constants. The behavior of parallel systems of first-order individual systems is summarized in Table 5.2.

The behavior presented in this section can cause some difficulty in terminology, since a stable overdamped system ( $\xi \geq 1$ ) is usually thought to have a monotonic response to a step input. This is true when the transfer function numerator is a constant, but it is not necessarily true when the numerator is a function of  $s$ . The potential dynamic behavior is summarized in Table 5.2.

<b>Poles</b>	<b>Response to nonperiodic input</b>	<b>Monotonic response to step</b>
Complex	Periodic	Not possible
Real	Nonperiodic	Possible, depends on numerator

TABLE 5.2

## Properties of parallel systems with first-order elements

Individual first-order systems	Parallel system
Each is first order	Order of the highest order in a parallel path
Each is stable	Stable, not periodic
Poles are $1/\tau_i$	Poles are $1/\tau_i, i = 1, \dots, n$
$t_{63\%}$	$t_{63\%} \neq \sum \tau_i$
Step response	Can be monotonic or experience overshoot or inverse response
Frequency response	Amplitude ratio can exceed steady-state process gain (for some frequency range)

The emphasis on complex dynamic responses in this section does not indicate that all systems with numerator zeros give unfavorable dynamics such as large overshoot or inverse response.

The engineer can analyze the physical process for possible parallel paths with different dynamics to identify potentially complex dynamics and then use quantitative methods to determine whether the behavior may cause difficulty for control. Each input must be considered separately, because the characteristics of the output dynamic response differ for different inputs.

## 5.5 ■ RECYCLE STRUCTURES

Recycle structures are used often in process plants, to return valuable material for reprocessing and to recover energy from effluent streams through heat exchange. Such interconnections, termed *process integration*, are often cited as potential causes of difficulty in plant operations in spite of their advantages in the steady state; therefore, it is important to understand the effects of recycle on process dynamics. This structure will be introduced through a process example and then will be generalized.

### EXAMPLE 5.3. Reactor with feed-effluent heat exchanger.

The process design shown in Figure 5.12 has a feed-effluent heat exchanger that can be used for a chemical reactor with a high feed temperature and a need for cooling the product effluent stream.

**Formulation.** The analysis begins with the transfer functions of the following individual input-output relationships, represented in the block diagram in Figure 5.13.

$$\frac{T_1(s)}{T_0(s)} = G_{H1}(s) = \frac{K_{H1}}{\tau_{H1}s + 1} \quad \frac{T_2(s)}{T_4(s)} = G_{H2}(s) = \frac{K_{H2}}{\tau_{H2}s + 1} \quad (5.49)$$

$$T_3(s) = T_1(s) + T_2(s) \quad \frac{T_4(s)}{T_3(s)} = G_R(s) = \frac{K_R}{\tau_{RS} + 1}$$

The block diagram shows the output of the reactor returning to influence an input

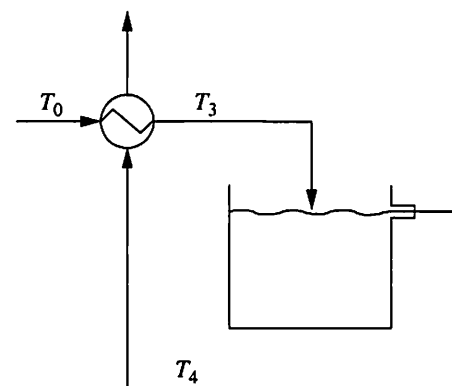


FIGURE 5.12

Reactor with feed-effluent heat exchanger in Example 5.3.

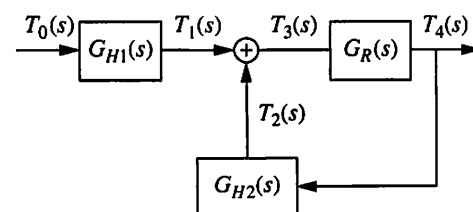


FIGURE 5.13

Block diagram of reactor-exchanger in Example 5.3.

to the reactor. This is feedback that has been introduced into the process by a recycle of energy. To determine the behavior of the integrated system, the overall input-output transfer function must be determined using block diagram algebra.

$$\begin{aligned} T_4(s) &= G_R(s)T_3(s) = G_R(s)[T_1(s) + T_2(s)] \\ &= G_R(s)[G_{H2}(s)T_4(s) + G_{H1}(s)T_0(s)] \end{aligned} \quad (5.50)$$

$$\frac{T_4(s)}{T_0(s)} = \frac{G_R(s)G_{H1}(s)}{1 - G_R(s)G_{H2}(s)}$$

It is immediately apparent from the overall transfer function that recycle has fundamentally changed the behavior of the system, because the characteristic polynomial in equation (5.50) has been influenced and the poles of the overall system are not the poles of the individual units. Thus, the stability of the overall system cannot be guaranteed, even if each individual system is stable!

To investigate the behavior of a recycle system further, models are defined for each of the individual processes in Figure 5.12. The following transfer functions are very simple, but the recycle system with these models experiences characteristics typical of realistic processes.

$$G_R(s) = \frac{3}{10s + 1} \quad G_{H1}(s) = 0.40$$

$$\begin{aligned} \text{With recycle:} \quad & G_{H2}(s) = 0.30 \\ \text{Without recycle:} \quad & G_{H2}(s) = 0 \end{aligned}$$

The gains are dimensionless ( $^{\circ}\text{C}/^{\circ}\text{C}$ ), and time is in minutes. The recycle heat exchanger model,  $G_{H2}(s)$ , represents the effect of the recycle stream temperature on the reactor inlet temperature. If no recycle existed, i.e., if the effluent did not exchange heat with the reactor feed,  $T_4(s)$  would have no effect on  $T_3(s)$ , so that  $G_{H2}(s)$  would not exist, which is represented by  $G_{H2}(s) = 0$ . These transfer function models can be substituted into equation (5.50) to determine the overall effect of a change in the process inlet temperature,  $T_0(s)$ , on the reactor temperature with and without recycle.

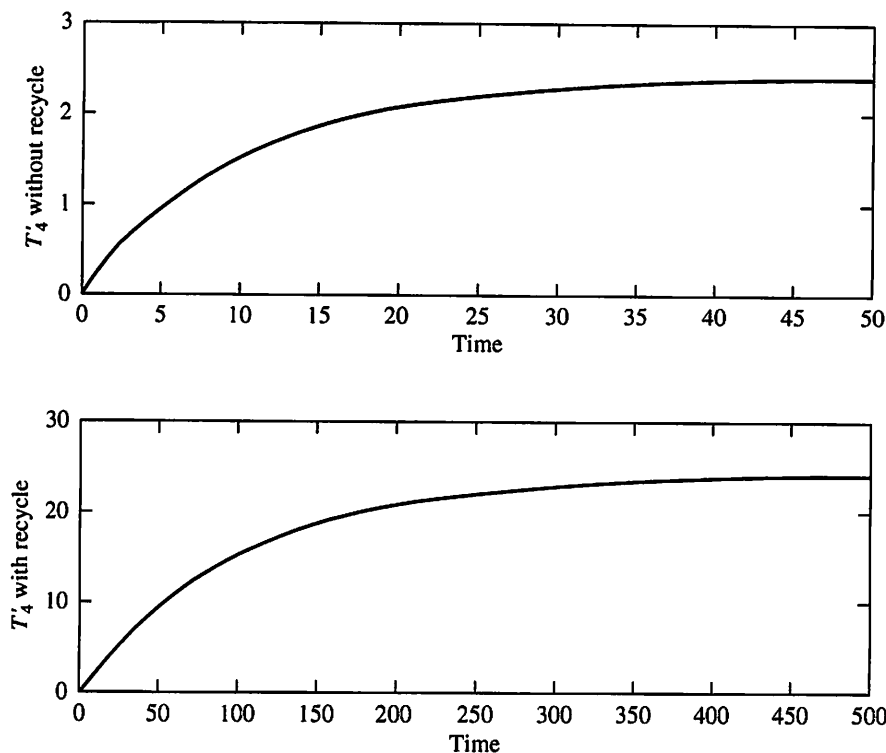
**With recycle:**

$$\frac{T_4(s)}{T_0(s)} = \frac{\left(\frac{3}{10s + 1}\right)(0.40)}{1 - \left(\frac{3}{10s + 1}\right)(0.30)} = \frac{12}{100s + 1} \quad (5.51)$$

**Without recycle ( $G_{H2}(s) = 0$ ):**

$$\frac{T_4(s)}{T_0(s)} = G_{H1}(s)G_R(s) = \frac{1.2}{10s + 1} \quad (5.52)$$

**Results analysis.** The foregoing expressions and the dynamic responses for a step input of  $2^{\circ}\text{C}$  in  $T_0$  in Figure 5.14 show the dramatic effect of recycle on the steady-state gain and time constant; both increase by a factor of 10 due to recycle. This change can be understood by analyzing the interaction between the exchanger and reactor in the recycle system during a transient; an increase in  $T_0$  causes an increase in  $T_3$  and then  $T_4$ , which causes an increase in  $T_2$ , which causes an increase in  $T_4$ , and so on; in short, the output change is reinforced through the recycle (feedback) exchanger. The system is still stable and self-regulatory, because of the dominant inherent negative feedback for the parameter values in this example, but the recycle has created an inherent *positive* feedback in the process,

**FIGURE 5.14**

**Dynamic responses for a 2°C step in  $T_0$  at time = 0 with and without recycle. (Note different scales.) Results from Example 5.3.**

which has significantly affected the dynamic response. The potentially unfavorable dynamic effects of recycle can be reduced through automatic control strategies, which retain most of the process performance benefits, as demonstrated for this chemical reactor design in Figure 24.11.

The simple example in this section demonstrates the potential effects of recycle on dynamic behavior:

1. Recycle can alter the stability and possibility for periodic behavior of the overall system, because it affects the poles of the overall system.
2. The time constants and steady-state gain of the overall system with recycle can be changed substantially from their values without recycle.

Again, understanding the effect of recycle on dynamic responses is an important aspect of process dynamics, and the material in this section is enhanced by reference to the studies of recycle in the Additional Resources at the end of this chapter.

## 5.6 □ STAGED PROCESSES

Staged processes are used widely in the process industries for multiple contacting of streams and can be considered as a special interconnection of elements, in which an element exchanges material and energy with only the adjoining stages.

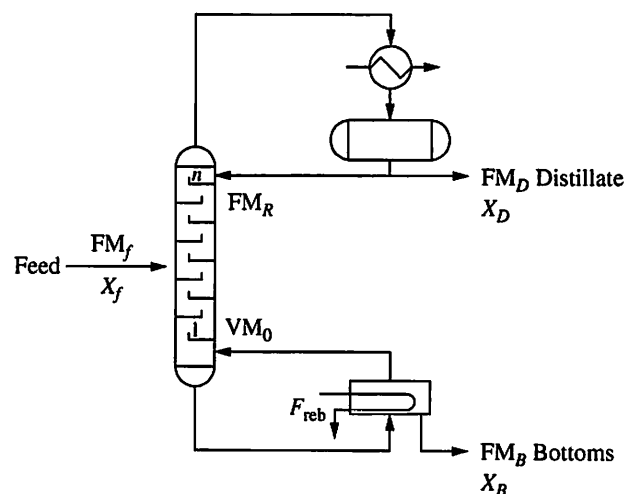
Some common examples are vapor-liquid equilibrium (Treybal, 1955), multieffect evaporation (Nisenfeld, 1985), and flotation (Narraway et al., 1991). Staged systems can experience a wide variety of dynamic behavior depending on the physical processes (e.g., mass transfer, heat transfer, and chemical reaction) that occur at each stage.

The fundamental model for a staged system must include all significant balances on every stage. However, the variables at every stage are not always of great importance for the overall performance of the process, because only the properties of the streams leaving the process are usually of interest. In some cases, a few intermediate variables could be important; an example is the flows on stages of a stripping tower, which might approach or exceed the hydraulic limits for proper contacting efficiency. We will assume in this section that the only output properties of interest are in the product streams.

In this section the dynamics of a distillation tower, shown in Figure 5.15, are considered as an example of staged systems to introduce the modelling approach and describe typical dynamic behavior. An accurate model of a multicomponent distillation tower must consider complex thermodynamic relationships and employ special numerical algorithms for the simultaneous solution of equilibrium expressions and material and energy balances. To simplify the presentation while maintaining a realistic model, the tower considered will separate only two components, and the phase equilibrium is assumed to be well represented by a constant relative volatility (Smith and Van Ness, 1987). Also, the energy balance at each stage can be simplified by the assumption of *equal molal overflow*, which implies that the heats of vaporization of both components are equal and mixing and sensible heat effects are negligible.

The assumptions are

1. The liquid level on every tray remains above the weir height.
2. Equal molal overflow applies.
3. Relative volatility  $\alpha$  and heat of vaporization  $\lambda$  are constant.
4. Holdup in vapor phase is negligible.



**FIGURE 5.15**  
**Distillation tower.**



The following nomenclature is used:

MM = molar holdup of liquid on tray

FM = molar flow rate of liquid

X = mole fraction of light component in liquid

$\lambda$  = heat of vaporization

VM = molar flow rate of vapor

Y = mole fraction of light component in vapor

The schematic of a general tray in Figure 5.16 shows that every tray has the potential for feed and product flows and heat transfer. With the assumptions and the general tray structure, the basic overall and component balances for each stage or tray ( $i = 1, \dots, n$ ) can be formulated as

Overall material (molar) balance on liquid phase:

$$\frac{dMM}{dt} = FM_{i+1} - FM_i + FM_{fi} - FM_{pi} - \frac{Q_i}{\lambda} \quad (5.53)$$

Quasi-steady-state overall material (molar) balance on vapor phase:

$$VM_i = VM_{i-1}^* - VM_{pi} + \frac{Q_i}{\lambda} \quad (5.54)$$

$$VM_{i-1}^* = VM_{i-1} + VM_{fi} \quad (5.55)$$

$$Y_{i-1}^* = \frac{VM_{i-1}Y_{i-1} + VM_{fi}Y_{fi}}{VM_{i-1}^*} \quad (5.56)$$

Light component balance on the tray:

$$\begin{aligned} \frac{d(MM_i X_i)}{dt} = & FM_{i+1} X_{i+1} + FM_{fi} X_{fi} - (FM_{pi} + FM_i) X_i \\ & - (VM_i + VM_{pi}) Y_i + VM_{i-1}^* Y_{i-1}^* \end{aligned} \quad (5.57)$$

This formulation is adequate for every equilibrium tray in the tower. For most trays, feed flows, product flows, and heat transferred are zero, while at least one tray has a nonzero feed. The top tray has a liquid feed, which is reflux, and its vapor stream goes to the total condenser. The bottom tray has its liquid go to the kettle reboiler, which is also an equilibrium stage. Note that although the equations can be formulated as shown, the computer implementation in this form would involve extensive multiplications for the zero streams; thus, an efficient implementation for a specific design would eliminate streams that are always zero.

Since there are many more variables than equations in the conservation balances, the model is not completely specified by these balances alone. The model requires constitutive expressions to relate liquid and vapor compositions. The phase equilibrium equation for a binary system with constant relative volatility  $\alpha$  is

$$Y_i = \frac{\alpha X_i}{1 + (\alpha - 1)X_i} \quad (5.58)$$

The model also requires constitutive expressions to relate liquid flows and inventories on the trays. The liquid flow from a tray is related to the level ( $L_i = MM_i / \rho_M A$ ) above the weir height,  $L_w$ , by (Foust et al., 1980)

$$FM_i = K_w \sqrt{\frac{MM_i}{\rho_M A} - L_w} \quad (5.59)$$

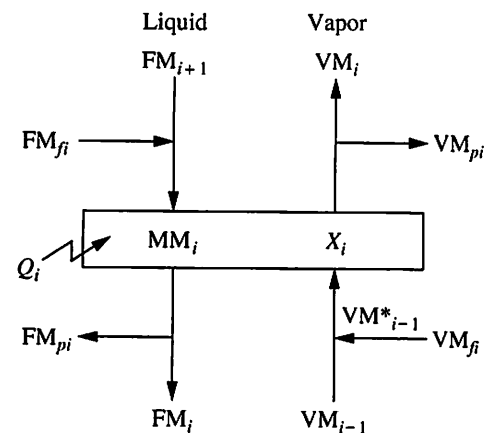


FIGURE 5.16

General tray used in modelling distillation.

with  $A$  being the cross-sectional area and  $\rho_M$  moles/m<sup>3</sup>. The modelling effort is not complete until models are developed for the associated equipment, which for this distillation tower includes the heat exchangers that vaporize part of the liquid accumulated in the bottom drum and condense the overhead vapor. The behavior of these is not particularly complex but requires feedback control to model properly. To maintain simple model structures without the need for control at this point, the reboiler duty is assumed to be proportional to the heating medium flow, and the vapor overhead is assumed to be completely condensed without subcooling, so that the pressure is maintained at a constant value by adjusting the condensing duty, thus

$$Q_{\text{cond}} = VM_n \lambda \quad (5.60)$$

$$Q_{\text{reb}} = K_{\text{reb}} F_{\text{reb}} \quad (5.61)$$

Also, the volumes in the overhead and bottom accumulators can be modelled by overall and component balances. In reality, the levels of these inventories would be controlled by adjusting the product flows; in this example, the levels are assumed exactly constant, so that the models become

$$FM_D = VM_n - FM_R \quad (5.62)$$

$$FM_B = FM_1 - VM_0 \quad (5.63)$$

The composition in the overhead accumulator ( $X_{n+1} = X_D$ ) can be determined from a component material balance:

$$MM_D \frac{dX_D}{dt} = VM_n Y_n - X_D (FM_D + FM_R) = VM_n (Y_n - X_D) \quad (5.64)$$

Again, with the inventory constant, the kettle reboiler can be modelled with a component material balance ( $X_0 = X_B$ ), equilibrium relationship, and a calculation of vapor flow based on heat transferred.

$$MM_B \frac{dX_B}{dt} = FM_1 X_1 - FM_B X_B - VM_0 Y_0 \quad (5.65)$$

$$Y_0 = \frac{\alpha X_B}{1 + (\alpha - 1) X_B} \quad (5.66)$$

$$VM_0 = \frac{Q_{\text{reb}}}{\lambda} \quad (5.67)$$

To specify the system completely, sufficient external input variables must be defined so that the degrees of freedom are zero. The feed flow and composition must be specified along with two additional variables, here selected to be the distillate product flow  $F_D$  and the reboiler heating flow  $F_{\text{reb}}$ . With these external variables specified, the degrees-of-freedom analysis summarized in Table 5.3 shows that the system is exactly specified. The number of equations is equal to the number of dependent variables; thus, there are zero degrees of freedom. Note that the parameters ( $\lambda$ ,  $\alpha$ ,  $K_w$ ,  $MM_D$ ,  $K_{\text{reb}}$ ,  $MM_B$ , and  $L_w$ ) were excluded from the analysis, because they are always constant. Also, the feed variables are determined by upstream process conditions. Typically, external variables like the reboiler heating flow rate and the distillate product flow rate are adjusted to achieve the desired product compositions; here, they are assumed known external variables. The model formulation included assumptions, like constant accumulator levels and pressure, that are not necessary but simplify the model and presentation.

**TABLE 5.3**  
**Distillation degrees of freedom for  $n$  trays**

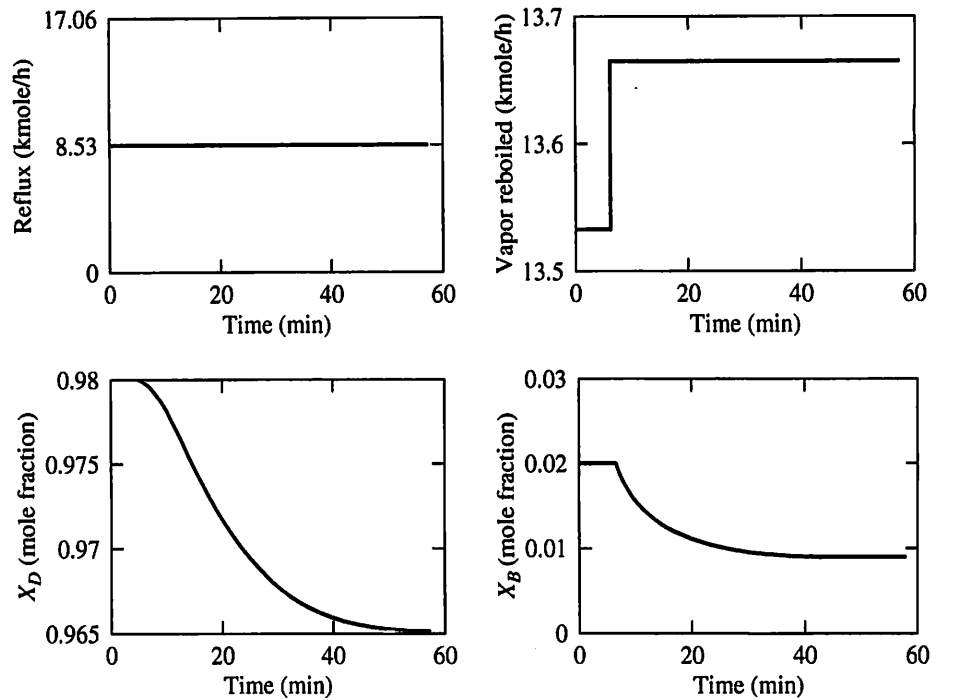
	<b>Equations</b>	<b>Variables (dependent)</b>	<b>External specified variables (independent)</b>
Trays	(5.53) to (5.59) for each tray  $(7n)$	MM, FM, VM, X, Y, Y*, V* for each tray plus $FM_{n+1}$ , $X_{n+1}$ , $V_0$ , $Y_0$  $(7n + 4)$	$FM_f$ , $X_f$ , $VM_f$ , $Y_f$ , $FM_p$ , $VM_p$ , $Q$ for each tray  $(7n)$
Overhead	(5.60), (5.62), and (5.64)  $(3)$	$Q_{\text{cond}}$  $(1)$	$FM_R$ or $FM_D$ , $MM_D$  $(2)$
Reboiler	(5.61), (5.63), (5.65), (5.66), and (5.67)  $(5)$	$X_B$ , $FM_B$ , and $Q_{\text{reb}}$  $(3)$	$F_{\text{reb}}$ , $MM_B$  $(2)$
Total	$7n + 8$	$7n + 8$	$7n + 4$

**TABLE 5.4**  
**Base case design parameters for example binary distillation**

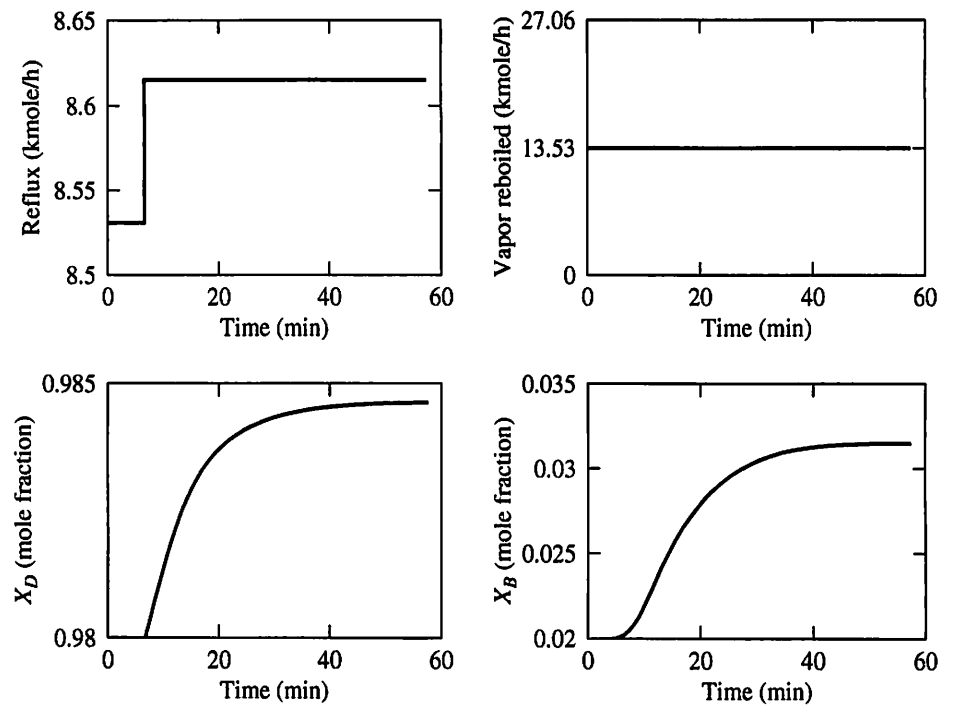
Relative volatility	2.4
Number of trays	17
Feed tray	9
Analyzer dead times	2 min
Feed light key	$X_F = 0.50$
Distillate light key	$X_D = 0.98$ fraction
Bottoms light key	$X_B = 0.02$ fraction
Feed flow	$FM_F = 10.0$ kmole/min
Reflux flow	$FM_R = 8.53$ kmole/min
Distillate flow	$FM_D = 5.0$ kmole/min
Vapor reboiled	$VM_0 = 13.53$ kmole/min
Tray holdup	$MM_i = 1.0$ kmole
Holdup in drums	$MM_B = MM_D = 10.0$ kmole

**EXAMPLE 5.4.**

Determine the dynamic behavior of a binary distillation tower with the parameters in Table 5.4. The model equations can be integrated numerically to determine the response of the system from specified initial conditions for any values or functions of the external variables. The dynamic responses are obtained by establishing a steady-state operating condition and introducing a single step change to one of the external variables; each step is 1 percent of the base case input value. (This is exactly how the experiment would be performed on the physical tower, as explained in Chapter 6.) The results are shown in Figure 5.17a and b. The composition responses are smooth monotonic sigmoidal curves, in spite of the complexity of the process. Note that changing a single input affects both



(a)



(b)

**FIGURE 5.17**

**Response of distillate and bottoms products in Example 5.6: (a) to reboiler step change; (b) to reflux step change. (These dynamic composition responses are obtained without sensor delays when the pressure and the distillate and bottoms accumulator levels are maintained constant.)**

product compositions—an important factor in subsequent control design as discussed in Chapters 20 and 21.

This summary presents a small sample of the results available on distillation dynamics. They have been presented as general guidelines for the behavior of two-product distillation with simple thermodynamics (e.g., no azeotropes) and no chemical reaction. The reader is encouraged to refer to the citations and Additional Resources for further details. This distillation example will be considered in later chapters, where the control of the product compositions, through adjustments to such variables as the reboiler duty and reflux flow, will be investigated.

## 5.7 □ MULTIPLE INPUT–MULTIPLE OUTPUT SYSTEMS

Many, but not all, of the systems modelled in Chapters 3, 4, and 5 have involved a single input and output. If intermediate variables existed, they could be eliminated using transfer functions and block diagram algebra to develop a *single input–single output* (SISO) equation. This approach helped to simplify our task of learning how to model dynamic responses and is applicable to some realistic processes. However, the majority of processes have several inputs, and process operation is concerned with more than one output simultaneously. For example, the nonisothermal chemical reactor in Section 3.6 has coolant flow and inlet concentration as inputs and reactor concentration and temperature as outputs. Also, the distillation tower in the previous section has distillate product flow, reboiler flow, and all feed properties and flow rate as inputs and concentration of both product streams as outputs.

The methods described in the previous two chapters for developing fundamental models—linearization, transfer functions, block diagrams—are all applicable to these *multiple input–multiple output* (MIMO) systems. Again, we see that many intermediate variables can exist in a process; in the distillation tower, the tray compositions and holdups are intermediate variables. These intermediate variables are included in the fundamental model and eliminated algebraically from the linearized input-output relationship.

### EXAMPLE 5.5.

Determine the dynamic response of the concentration in the CSTR with second-order reaction in Example 3.5 to step changes in the inlet concentration and the feed flow rate. The definitions of the changes are

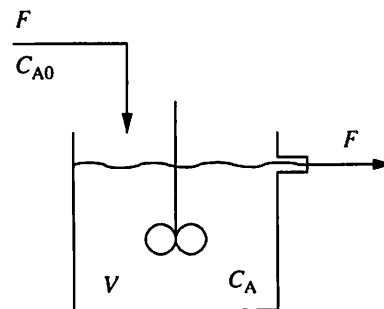
$$\text{Feed concentration step: } \Delta C_{A0} = 0.0925 \text{ mol/m}^3 \quad \text{at } t = 2 \text{ min}$$

$$\text{Feed flow rate step: } \Delta F = -0.0085 \text{ m}^3/\text{min} \quad \text{at } t = 7 \text{ min}$$

The effect of several input variables on a single output variable can be determined through the individual input-output models. The fundamental model for the reactant component material balance is repeated here:

$$V \frac{dC_A}{dt} = F(C_{A0} - C_A) - V k C_A^2 \quad (5.68)$$

To clarify the linearity of the model, all constants are substituted in equation (5.68)



to give

$$(2.1) \frac{dC_A}{dt} = F(C_{A0} - C_A) - (2.1)(0.040)C_A^2$$

The model is nonlinear because of the product of variables and the concentration terms. The model in equation (5.68) can be linearized for a change in the inlet concentration (with flow constant) or for a change in the feed flow (with inlet concentration constant), giving

$$\tau_{CA0} \frac{dC'_A}{dt} + C'_A = K_{CA0} C'_{A0} \quad (5.69)$$

$$\tau_F \frac{dC'_A}{dt} + C'_A = K_F F' \quad (5.70)$$

with

$$\tau_{CA0} = \frac{V}{F_s + 2VkC_{As}} \quad K_{CA0} = \frac{F_s}{F_s + 2VkC_{As}}$$

$$\tau_F = \frac{V}{F_s + 2VkC_{As}} \quad K_F = \frac{(C_{A0s} - C_{As})}{F_s + 2VkC_{As}}$$

These two models can be solved for step changes to give

$$[C'_A(t)]_{CA0} = \Delta C_{A0} K_{CA0} (1 - e^{-t_1/\tau_{CA0}}) \quad \text{with } t_1 = t - 2 > 0 \quad (5.71)$$

$$[C'_A(t)]_F = \Delta F K_F (1 - e^{-t_2/\tau_F}) \quad \text{with } t_2 = t - 7 > 0 \quad (5.72)$$

Note that the times from the steps are represented by different symbols ( $t_1$  and  $t_2$ ) because the two step changes are introduced at different times; also, the reactant concentration change is zero until  $t_1 > 0$  or  $t_2 > 0$ , respectively. The total change in reactor concentration of A is the sum of the changes due to inlet concentration and flow.

$$C_A(t) = C_{As}(t) + [C'_A(t)]_{CA0} + [C'_A(t)]_F \quad (5.73)$$

For the data in Example 3.5, the following values can be determined:

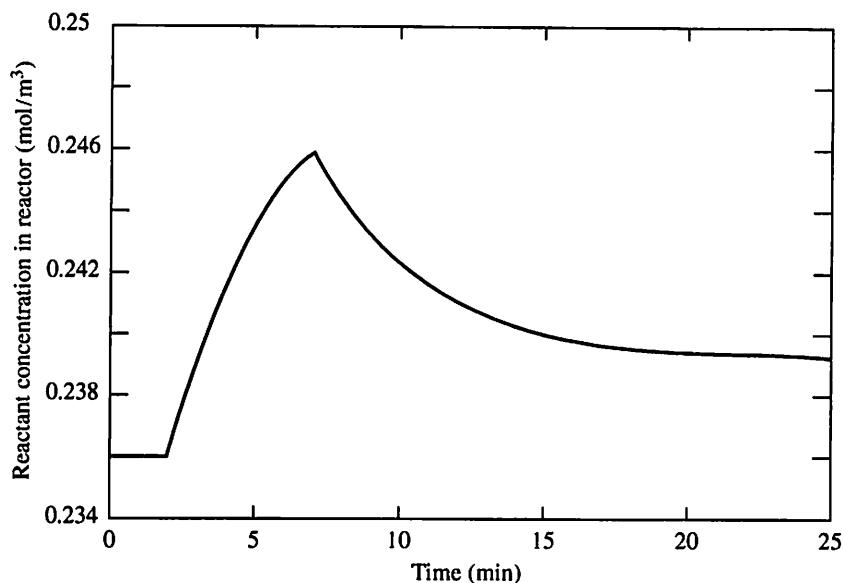
$$\begin{array}{lll} V = 2.1 \text{ m}^3 & F_s = 0.085 \text{ m}^3/\text{min} & k = 0.50 \text{ [(mole/m}^3\text{)min]}^{-1} \\ C_{A0s} = 0.925 \text{ mole/m}^3 & C_{As} = 0.236 \text{ mole/m}^3 & \tau_{CA0} = 3.62 \text{ min} \\ K_{CA0} = 0.146 & \tau_F = 3.62 \text{ min} & K_F = 1.19 \text{ (mol/m}^3\text{)/(m}^3\text{/min)} \end{array}$$

The results from the linearized analysis in equations (5.71) to (5.73) are given in Figure 5.18. Clearly, the output concentration is the sum of two first-order step responses beginning at different times. This modelling approach can be extended to any number of input variables affecting an output.

### EXAMPLE 5.6.

Sketch a block diagram showing the relationship between the input variables, reflux flow and reboiled vapor, and the output variable, light component mole fraction in the distillate and bottoms products.

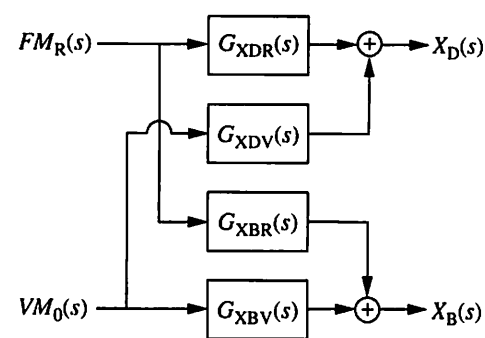
The data in Figure 5.16 show that both input variables affect both output variables. Thus, each input has two transfer functions, one for each of the output variables. The sketch for this process is shown in Figure 5.19. A natural question is "How are the transfer functions determined?" In previous examples, the



**FIGURE 5.18**

**Dynamic response of reactant concentration for a step increase in inlet concentration ( $t = 2$ ) and step decrease in flow rate ( $t = 7$ ) in Example 5.5.**

fundamental model has been linearized and all intermediate variables eliminated by algebraic manipulations. However, the fundamental model for the distillation process is large, involving about 150 equations, so that the analytical procedure would be excessively time-consuming. Fortunately, the transfer functions can be determined experimentally from data very similar to Figure 5.16, and this empirical modelling procedure is explained in the next chapter.



**FIGURE 5.19**

**Block diagram for the linearized models for a two-product distillation process.**

## 5.8 ■ CONCLUSIONS

The results of this chapter clearly demonstrate that process structures have strong effects on dynamic behavior and that these effects can be predicted using the methods presented in the previous chapters. Many of the strongest results relate to the “long-time” behavior of the systems, because they are determined by the poles of the transfer function and are independent of the numerator zeros. These properties involve stability and the related tendency for over- or underdamped behavior. However, the numerators also play an important role in the dynamic response, as shown by the examples in the section on parallel structures.

It is worth noting that each of these process structures is covered individually to clarify the analysis of their effects on dynamic behavior. Naturally, a process may contain several of these structures, all of which will influence its behavior. The study of complex processes is delayed until Parts V and VI, which address the control of multiple input–multiple output systems.

Finally, in the last three chapters, dynamic responses of many processes to a step input have been shown to have a sigmoidal shape. This means that these processes could be approximated by adjusting parameters in a model of simple

structure. While this observation is not especially helpful for analytical modelling, it is very important for empirical modelling, which develops models based on experimental data. This is the topic of the next chapter.

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- Weber, T., *An Introduction to Process Dynamics and Control*, Wiley, New York, 1973.

## ADDITIONAL RESOURCES

Recycle systems occur frequently and substantially affect process dynamics. Some studies on these effects are noted here.

- Douglas, J., J. Orcutt, and P. Berthiaume, "Design of Feed-Effluent Exchanger-Reactor Systems," *IEC Fund.*, 1, 4, 253–257 (1962).
- Gilliland, E., L. Gould, and T. Boyle, "Dynamic Effects of Material Recycle," *Joint Auto. Cont. Conf.*, Stanford, CA, 140–146 (1964).
- Luyben, W., "Dynamics and Control of Recycle Systems. 1. Simple Open-Loop and Closed-Loop Systems," *IEC Res.*, 32, 466–475 (1993).
- Rinard, I., and B. Benjamin, "Control of Recycle Systems, Part 1. Continuous Systems," *Auto. Cont. Conf.*, 1982, WA5.

Inverse response can be a vexing problem for control. The engineer should understand the process causes of inverse response systems and modify the design to mitigate the effect.

- Iionya, K., and R. Altpeter, "Inverse Response in Process Control," *IEC*, 54, 7, 39 (1962).

Modelling complex distillation columns is a challenging task that has received a great deal of study.

- Fuentes, C., and W. Luyben, "Control of High Purity Distillation Columns," *IEC (Industrial and Engineering Chemistry), Proc. Des. Devel.*, 22, 361–366 (1983).
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Formulating models and programming numerical solution methods is always a good learning opportunity; however, the task is time-consuming. The use of commercial simulation systems is recommended for modelling complex processes. These systems can simulate the dynamics of typical chemical processes using standard models and accurate physical property relationships.

- Aspen, Aspen Dynamics™ and Aspen Custom Modeller™, Aspen Technology, 10 Canal Park, Cambridge, MA, 1999.
- HYSYS.PLANT v2.0 Documentation, Dynamic Modelling Manual, Hyprotech Ltd., Calgary, 1998.

The guidance before the questions in Chapters 3 and 4 is appropriate here as well. The key new issue introduced in this chapter and demonstrated in these questions is the effect of structure on the behavior of relatively simple individual elements.

## QUESTIONS

- 5.1. A linearized model for a stirred-tank heat exchanger is derived in Example 3.7 for a change in the coolant flow rate. Extend these results by deriving the model for simultaneous changes in the coolant flow rate and inlet temperature. Also, determine an analytical expression for the outlet temperature  $T'(t)$ , for simultaneous step changes in the coolant flow and inlet temperature. (You may use all results from Example 3.7 without deriving.)
- 5.2. The jacketed heat exchanger in Figure Q5.2 is to be modelled. The input variable is  $T_0'$ , and the output variable is  $T'$ . The inlet coolant temperature is constant. The following assumptions may be made:
- (1) Both vessels are well mixed.
  - (2) Physical properties are constant.
  - (3) Flows and volumes are constant.

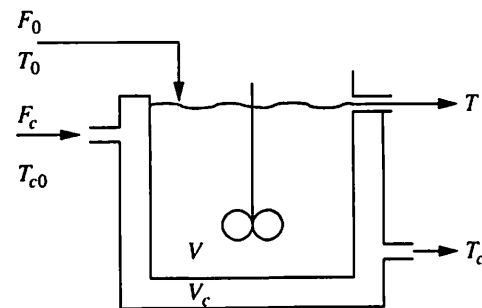
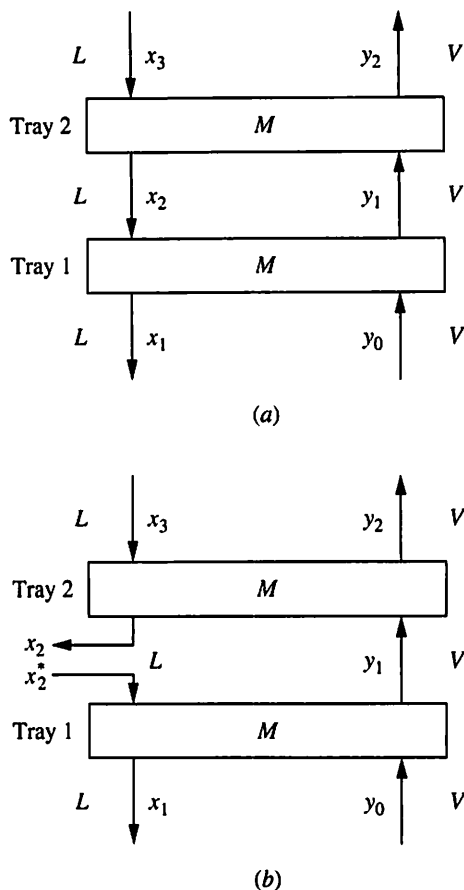


FIGURE Q5.2



**FIGURE Q5.3**

$$(4) Q = UA(T - T_c)$$

(5) The dynamic balances on *both* volumes must be solved simultaneously.

- Write the basic balances for both volumes in deviation variables.
- Take the Laplace transforms.
- Combine into the transfer function  $T'(s)/T'_0(s)$ .
- Analyze this result to determine whether the dynamic behavior is (i) stable and (ii) periodic. Remember that these properties are defined by the denominator of the transfer function.
- The transfer function ignores initial conditions of the system. Briefly explain why the transfer function is useful—in other words, what properties can be determined easily using the transfer function?

**5.3.** The continuous-time systems of two stages shown in Figure Q5.3a and b are to be analyzed. Assumptions are the following:

- Liquid holdups are constant =  $M$ .
- Constant molal overflow; the liquid ( $L$ ) and vapor ( $V$ ) flows are constant.
- The concentrations  $x_3$  (and  $x_2^*$  in Figure Q5.3b) are constant.
- The accumulation in the vapor phase is negligible.
- Equilibrium can be modelled as  $y_i = Kx_i$  for this binary system.

The nature of the dynamic behavior is to be determined for the input-output  $x_2(s)/y_0(s)$ .

- Derive the time-domain equations describing the dynamics of the concentrations on the two trays,  $x'_1(t)$  and  $x'_2(t)$ , to the input variable  $y'_0(t)$ , in deviation variables.
- Combine the results of (a) into the single transfer function  $x_2(s)/y_0(s)$ .
- Determine the nature of the response. Is it (i) stable, (ii) over- or underdamped?
- Is the response of  $x_2$  to a step change in  $y_0$  in Figure Q5.3a faster or slower than in the system in Figure Q5.3b (with the same parameter values and  $x_2^*$  constant)?

**5.4.** The series of four chemical reactors are shown in Figure Q5.4. Each reactor is constant volume and constant temperature, and the flow rate is constant. The reaction is  $A \rightarrow B$  with the rate expression  $r_A = -kC_A$ . The concentration of component A in the last reactor is to be controlled, and the feed concentration of the inlet to the first reactor is a potential manipulated variable.

- Derive the model (algebraic and differential equations) relating  $C_{A0}$  to  $C_{A4}$ .
- Combine these equations into one input-output model that has only  $C_{A0}$  and  $C_{A4}$ , with other relevant variables eliminated. (Hint: Taking the Laplace transform of the equations in deviation variables is a good approach.)
- Based on the model in (b), determine
  - The order of the system
  - The stability of the system
  - The damping of the system

- (iv) The gain of the system  
 (v) The shape of the response of  $C_{A4}$  to a step in  $C_{A0}$
- (d) Based on your results in (c), does a causal relationship exist between  $C_{A0}$  and  $C_{A4}$ ?
- (e) Based on your results in (d), is it possible to control  $C_{A4}$  by adjusting  $C_{A0}$ ?

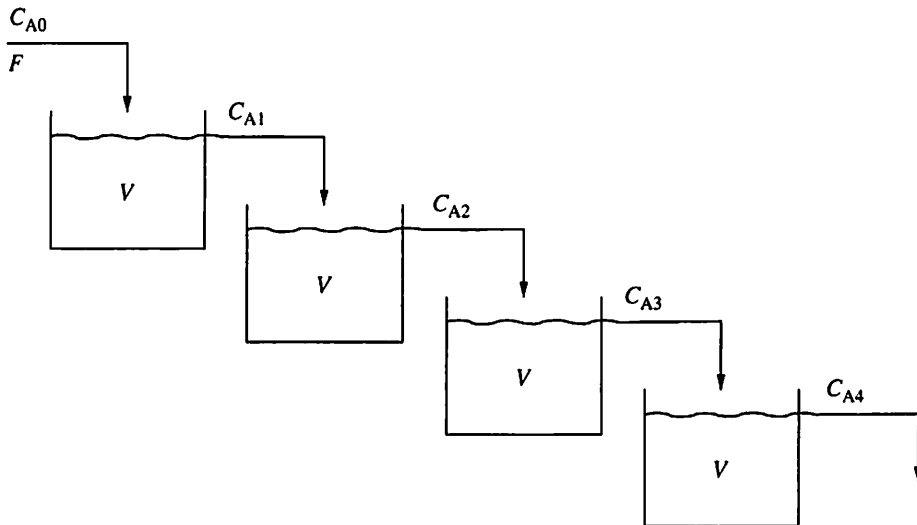


FIGURE Q5.4

Series stirred-tank reactors.

5.5. The recycle mixing system in Figure Q5.5 is to be considered. The feed flow is 1 unit, and the recycle flow is 9 units. The pipe has a dead time of 10 seconds, and the recycle has negligible dynamics. The system is initially at steady state with pure solvent entering as feed. At time = 0, the concentration of the feed increases to 10%A. Plot the concentration at the exit of the pipe from  $t = 0$  to the new steady state.

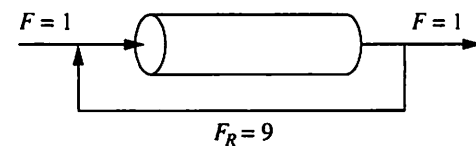


FIGURE Q5.5

5.6. The chemical reactor without control of temperature or concentration in Figure Q5.6 is to be modelled and analyzed. The assumptions are as follows:

- (1)  $C_p$  ( $C_p = C_v$ ), density,  $UA$  are constant.
- (2)  $Q = UA(T - T_{cin})$
- (3)  $F$ ,  $T_c$ ,  $T_0$ , level are constant.
- (4) Disturbance is  $C_{A0}(t)$ .
- (5) Heat of reaction is significant.
- (6) Heat losses are insignificant.
- (7) System is initially at steady state.
- (8) Rate of reaction =

$$-r_A = k_0 e^{-E/RT} C_A \frac{\text{mole}}{(\text{m}^3)(\text{min})}$$

- (a) Derive the material and energy balances for this reactor. Carefully define the system, state all assumptions, and show all steps, especially in the energy balance.

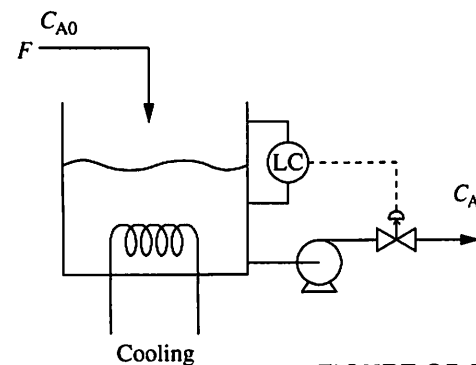


FIGURE Q5.6

- (b) Linearize the equations about their steady-state values and express them in deviation variables.
- (c) Based on the linearized equations, state whether the system can experience overdamped behavior, and state mathematical criteria as a basis for your decision. (Hint: Solve for the terms that affect the exponents of the dynamic response, and establish criteria for the qualitative characteristics.)
- (d) Repeat (c) for underdamped behavior.
- (e) Repeat (c) for unstable behavior.

**5.7.** A single isothermal CSTR has the following elementary reactions.



Only component A is in the feed stream, and its concentration,  $C_{A0}$ , can change as the input to the system. Answer the following questions for both Cases I and II.

- (a) Derive the model describing the concentration of component B in the reactor.
- (b) Which of the general system structures covered in this chapter describes this system?
- (c) Determine whether the system can experience underdamped, overdamped, and unstable behavior for physically possible parameter values.
- (d) Describe the response of this system to feed concentration step changes in  $C_{A0}$  and determine which system would have a faster response.
- (e) Repeat all parts of this question, with the composition of A in the reactor being the output variable.

**5.8.** Figure 5.1 can be expanded to include more process systems and more inputs.

- (a) Include the following systems, with a sketch of a physical process: (1)  $1/(\tau s + 1)^3$  and (2)  $e^{-\theta s}/(\tau s + 1)$ .
- (b) Include the following inputs for all systems: (1) ramp ( $Ct$ ) and (2) pulse of finite duration.

**5.9.** The dynamic response of  $T_6$  in the heat exchanger and stirred-tank system in Figure Q5.9 is to be determined for a step increase in the flow to the exchanger  $F_{ex}$ , with the total coolant flow  $F_c$  constant. (Assume that negligible transportation lag occurs in the pipes.)

- (a) Derive the models for both stirred tanks.
- (b) Determine the individual transfer functions.
- (c) Derive the overall transfer function.
- (d) Which of the general system structures covered in this chapter describes this system?
- (e) Explain the numerator zeros (if any) and poles in the system.
- (f) Describe the dynamic response of this system for the input step change in  $F_{ex}$ .

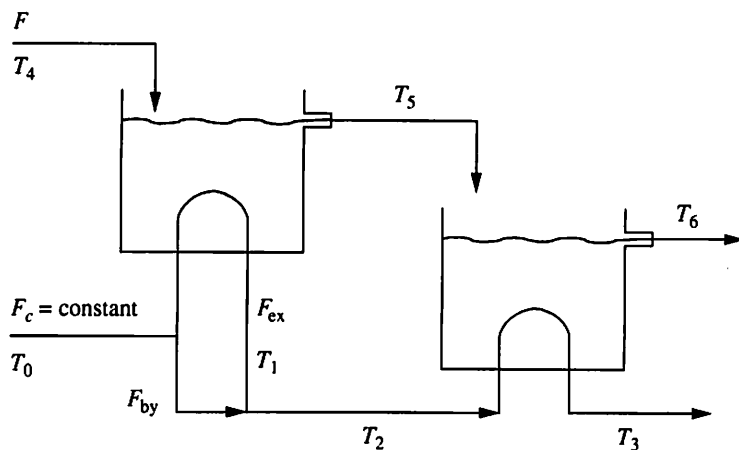


FIGURE Q5.9

5.10. The system of vessels in Figure Q5.10 has gas flowing through it, and  $F_0$  is independent of  $P_1$ .

- (a) Assume that the flow through the restrictions is subsonic.
- (1) Derive linearized models for the pressure in each system.
  - (2) Determine the transfer function for  $F_2(s)/F_0(s)$ .
  - (3) Describe the response of this system to a step in  $F_0$ .
- (b) Repeat the analysis in part (a) for sonic flow through the restrictions.

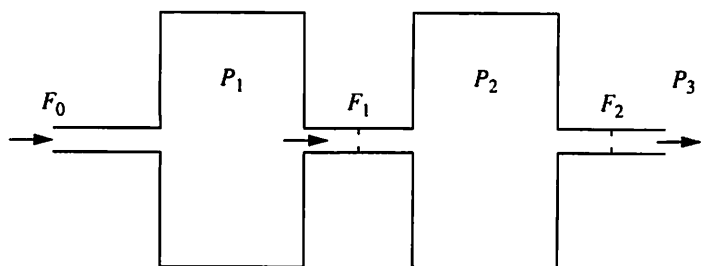


FIGURE Q5.10

5.11. Answer the following questions.

- (a) Demonstrate that the dynamic behavior of a series of stable, first-order systems approaches the dynamic behavior of a dead time as the number of first-order systems becomes large, with  $\tau_n = \tau_1/n$ . Determine the value of the dead time.
- (b) For the reactor with recycle in Example 5.5, determine the value of the heat exchanger gain,  $K_{H2}$ , that would cause the system to be unstable. Explain the expected dynamic response to an increase in the feed temperature.
- (c) Discuss the manual control of a series of noninteracting time constants, a parallel system with overshoot, and a parallel system with inverse response. What would be your thought process for feedback control?

(d) What would be the order of the transfer function between the input  $FM_D$  and the output  $X_D$  for the distillation tower in Section 5.6?

**5.12.** An autocatalytic system has a chemical reaction in which the product influences the rate; such kinetics occur in biological systems. Consider the following system occurring in a constant-volume, isothermal, well-stirred reactor.



- (a) Formulate a dynamic model of the reactor to predict the concentration of B in the reactor.
- (b) Determine the possible steady-state values for  $C_B$  when only A is present in the feed. (Hint: Two possible steady states exist.)
- (c) Under what conditions does the reactor go to each steady state?
- (d) Reformulate the model and answer all questions for the case in which the product is separated and some pure B is returned to the reactor as a recycle. What would be the advantage of this recycle? How would the recycle affect the gain and time constant of  $C_B$  in response to a change in  $C_{A0}$ ?
- 5.13.** For each of the systems in Figure Q5.13, demonstrate through a fundamental model whether the system inventory is self-regulating or not for changes in flow in. In all cases, the flow in ( $F_{in}$ ) can change independent of the inventory in the vessel.
- (a) A heat exchanger in which the pure-component liquid entering at its boiling point in the vessel boils and the duty is proportional to the heat transfer area.
- (b) A liquid-filled tank with a constant flow out.
- (c) A gas-filled system with a moving roof and a constant mass on the roof. The gas exits through a partially open restriction.
- (d) A gas-filled system with constant volume. The gas exits through a partially open restriction.
- 5.14.** The stirred-tank mixing process in Figure Q5.14 is to be analyzed. The system has a single feed, two tanks, and a single product. All flow rates, along with the levels, are constant. Answer the following questions completely. You may assume that (1) the tanks are well mixed, (2) the density is constant, and (3) transportation delays due to the pipes are negligible. For parts (a) through (c),  $F_3 = F_0$ .
- (a) Derive the analytical model for the input-output system  $C_{A0}$  and  $C_{A2}$  with all flows constant.
- (b) What is the general structure of the system in (a)?
- (c) What conclusions can be determined for the system in (a) regarding the stability, periodicity, and either overshoot or inverse response for a step input?
- (d) Determine the answers for (a) through (c) for (i)  $F_3 = 0$  and (ii)  $F_3 =$  very large.
- 5.15.** The system in Figure Q5.15 has two stirred tanks; the first is a heat exchanger, and the second is a CSTR. The product of the reactor exchanges

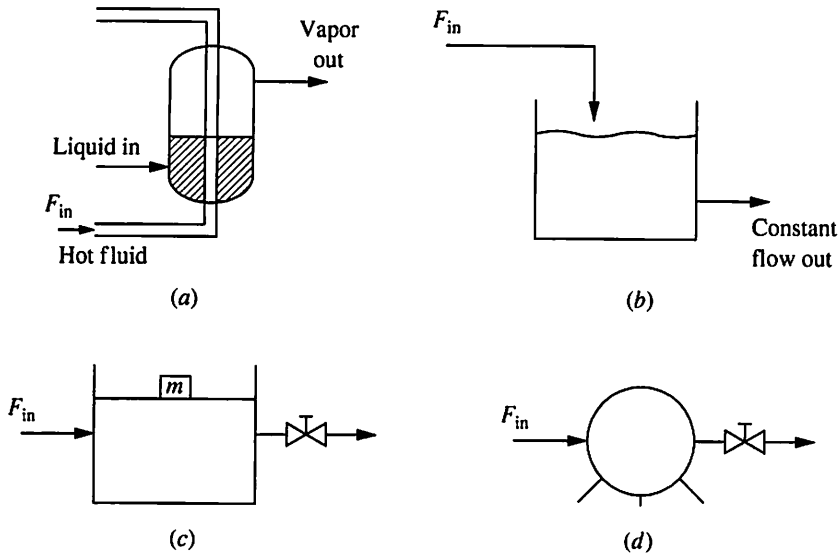


FIGURE Q5.13

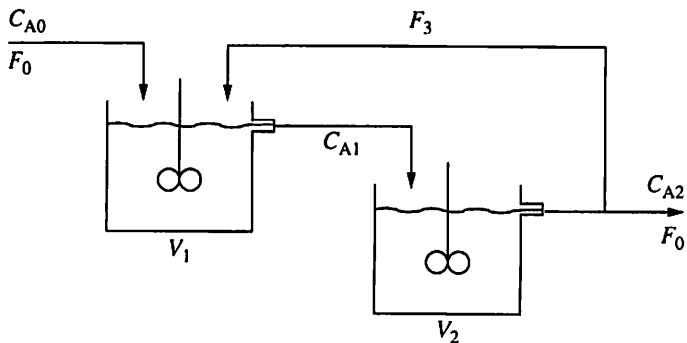


FIGURE Q5.14

heat with the feed in the heat exchanger. A single, zeroth-order reaction of  $A \rightarrow$  products occurs in the second reactor with a heat of reaction  $(-\Delta H_{rxn})$ .

- Formulate a model of the system to predict the temperature response in both tanks to a change in the feed temperature with all flows constant, and linearize the model. Determine to which process structure category this process belongs.
- Determine under what conditions the system would experience (i) periodic behavior and (ii) unstable behavior.
- Discuss your results and limitations in the model.

[Hint: This system is simpler than Example 3.10, in that the coolant flow is constant; thus,  $UA = aF_c^b$  is constant. It is more complex in that the energy balances for the two tanks must be solved simultaneously.]

- 5.16.** The recycle system in Figure Q5.16 has a well-mixed, isothermal, constant-volume reactor and subsequent separation unit, in which the unreacted feed is separated from the product and returned to the reactor. A single step change occurs in the reactor temperature, which can be considered a step in the rate constant of the first-order reaction. Model the system and determine and compare the dynamics for two operating methods.

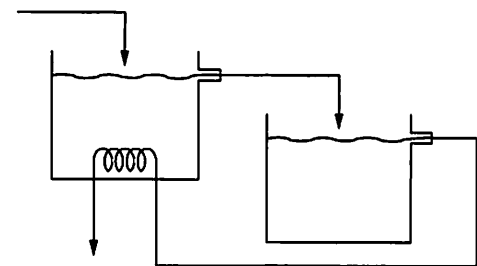
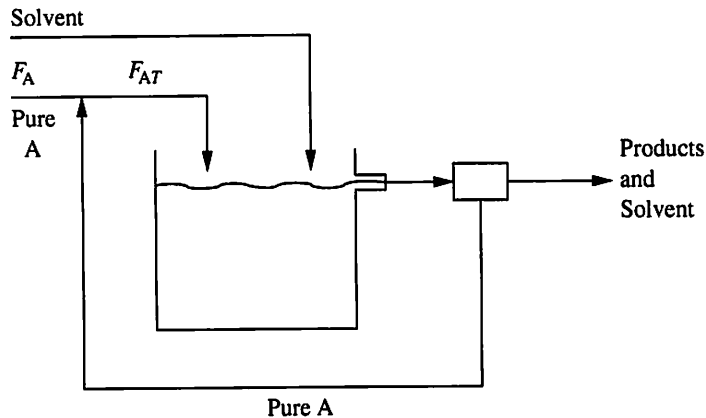


FIGURE Q5.15

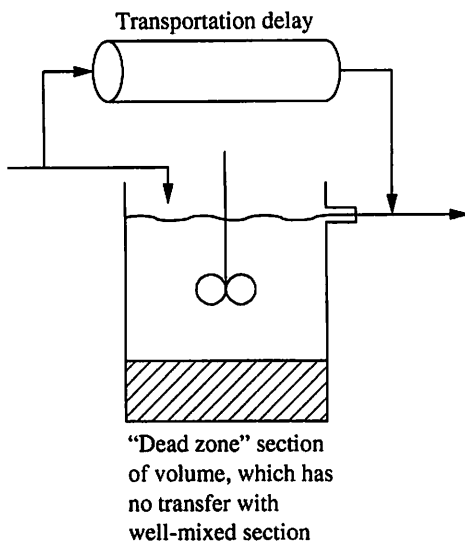


**FIGURE Q5.16**

- (a) The flow  $F_A$  is constant.  
 (b) The flow  $F_{AT}$  is constant.

**5.17.** A tubular heat exchanger with plug flow in the tube has steam at a constant temperature on the shell side. The system is initially at steady state with no temperature driving force, and the steam is introduced in a step to the shell.

- (a) Determine the tube outlet temperature as a function of time. This will require analyzing a distributed-parameter model.  
 (b) Formulate a lumped-parameter model that would give an approximate result for the tube outlet temperature.



**FIGURE Q5.18**

**5.18.** One way to account for imperfect mixing in a single stirred tank is to include commonly occurring *nonidealities* and fit parameters in a model to empirical data. For the nonideal model in Figure Q5.18, plot the shapes of the step and impulse responses for various values of the nonidealities. Could you fit an imperfect model using one of these sets of data?

**5.19.** Derive the models reported in Figures 5.2 and 5.3 for the electrical and mechanical systems.

**5.20.** From the principles in this chapter (and Appendix D), estimate the shape and  $t_{63\%}$  of the step change for the following systems: (a) Example 3.3, (b) Example 3.10, (c) Question 4.15, and (d) Question 4.18.

**5.21.** A nonisothermal CSTR with heat transfer is modelled in Section C-2 in Appendix C. For each of the following situations, describe the possible shapes of the dynamic response of the concentration,  $C_A$ , to a step change in the coolant flow rate. There may be more than one per situation. Explain your answers by discussing, for example, the interaction between the material and energy balances.

- (a) No chemical reaction,  $k_0 = 0$   
 (b) Nonzero chemical reaction, but  $\Delta H_{rxn} = 0$   
 (c) General case with nonzero reaction and heat of reaction